CoqPilot

a plugin for LLM-based generation of proofs

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Nikita Khramov

Anton Podkopaev

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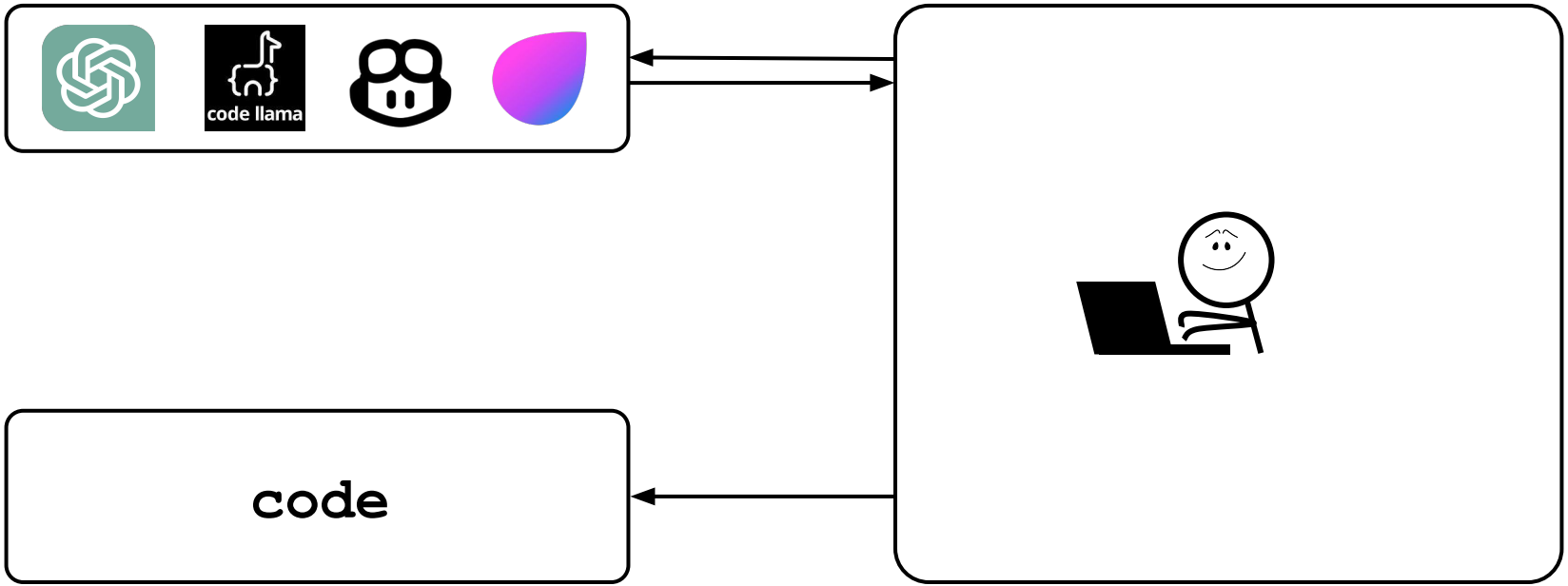
LLMs are used more and more for code generation



Do Users Write More Insecure Code with AI Assistants?

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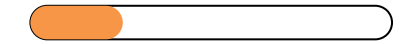
introduce security ***vulnerabilities***



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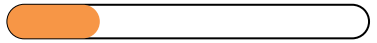
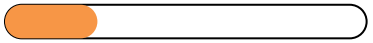


rate their ***insecure*** code ***as secure***



Generate a list sorting function in Python without using any library functions

```python
def custom_sort(arr):
    n = len(arr)
    for i in range(n):
        # Last i elements are already sorted
        for j in range(0, n - i - 1):
            # Traverse the array from 0 to n-i-1
            # Swap if the element found is greater
            # than the next element
            if arr[j] > arr[j + 1]:
                arr[j], arr[j + 1] = arr[j + 1], arr[j]
    return arr
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Testing may **not** be **sufficient** (eg, concurrency)

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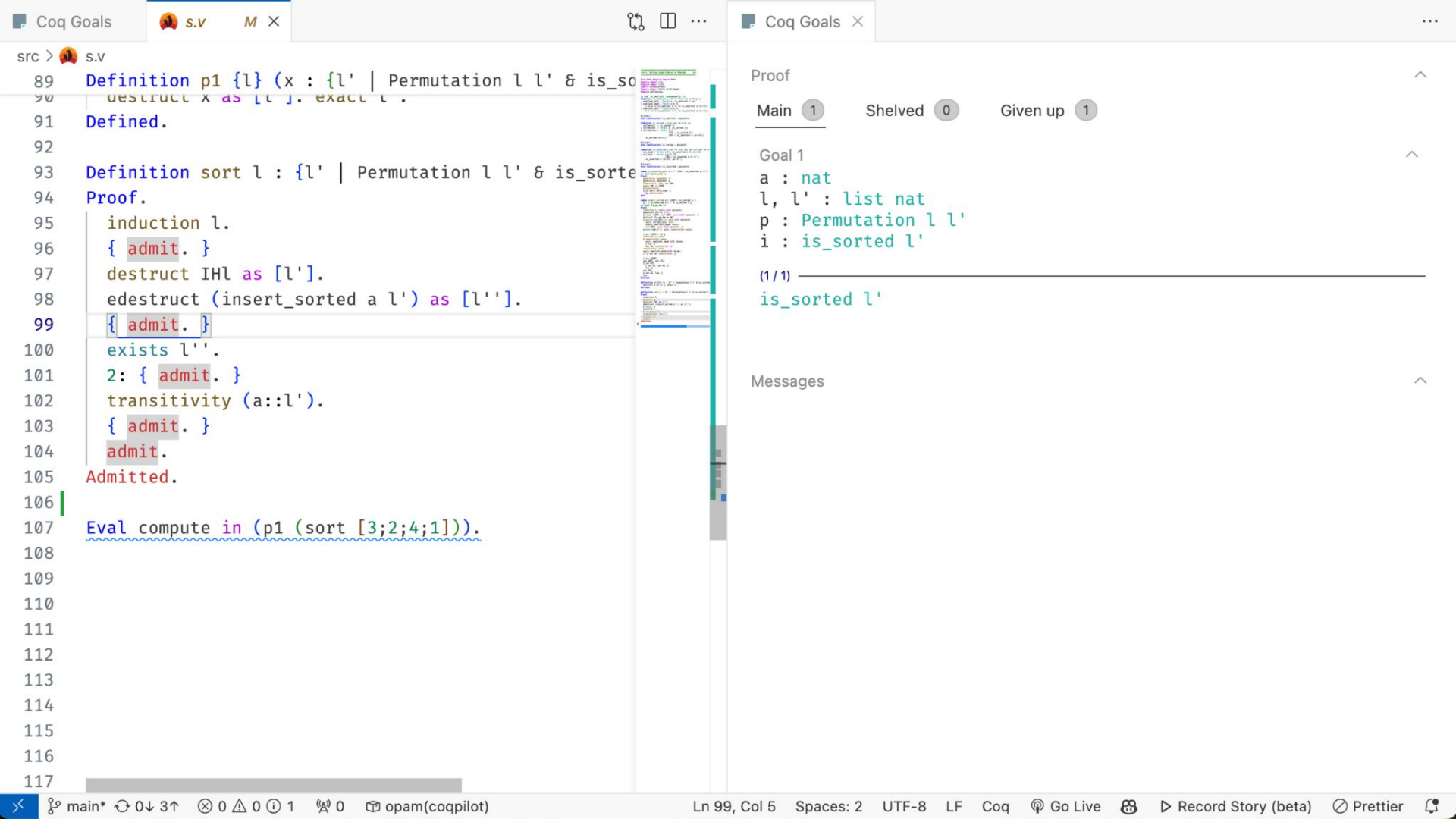
Plain **English** is **hard** to debug and **imprecise**

CoqPilot


Coq Goals
s.v
M
src
s.v
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destruct x as [l']. exact l'.
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Proof.
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0↓ 3↑
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opam(coqpilot)
Ln 99, Col 5
Spaces: 2
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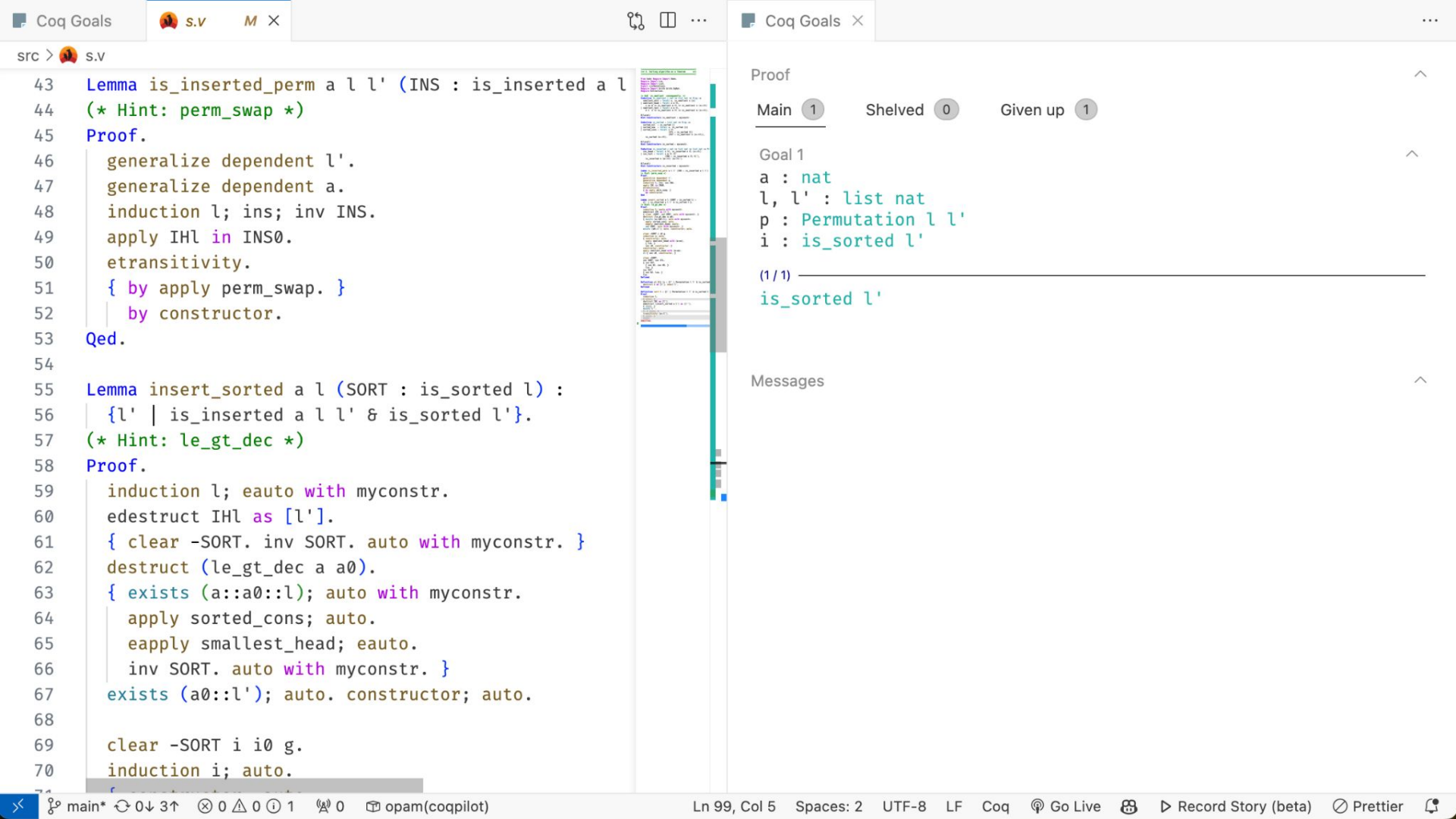
Coq
Go Live
Record Story (beta)
Prettier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Lemma is_inserted_perm a l l' (INS : is_inserted a l
(* Hint: perm_swap *)
Proof.
generalize dependent l'.
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induction l; ins; inv INS.
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Lemma insert_sorted a l (SORT : is_sorted l) :
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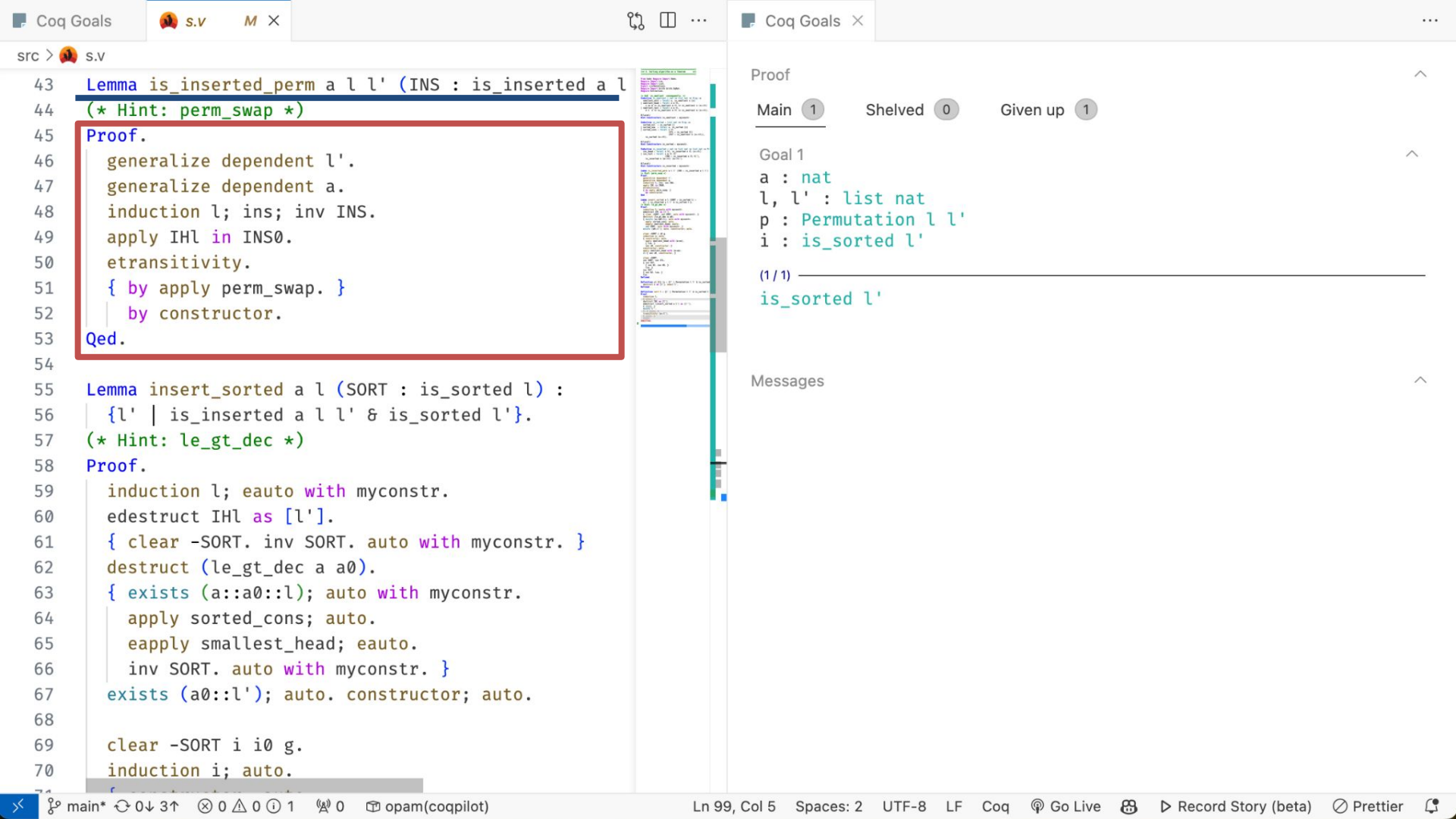
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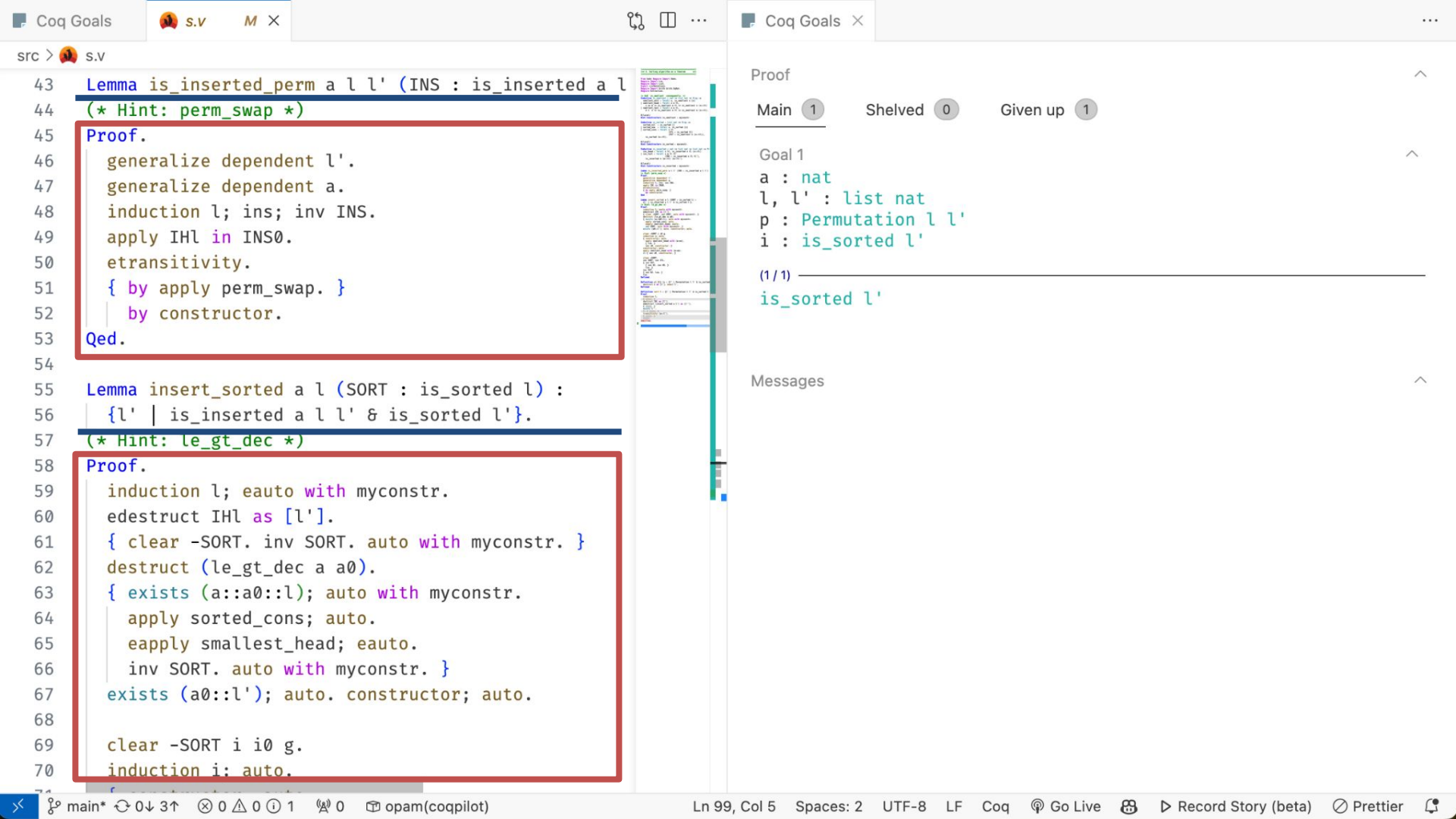
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s.v M

src > s.v

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Admitted.

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Coq Goals

Proof

Not in proof mode

Messages

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main* 0↓ 3↑ Launchpad ⊗ 0 ⚠ 0 ⓘ 1 0 opam(coqpilot) vscoq-language-server 2.1.0, coq 8.19.0 Go Live Record Story (beta)

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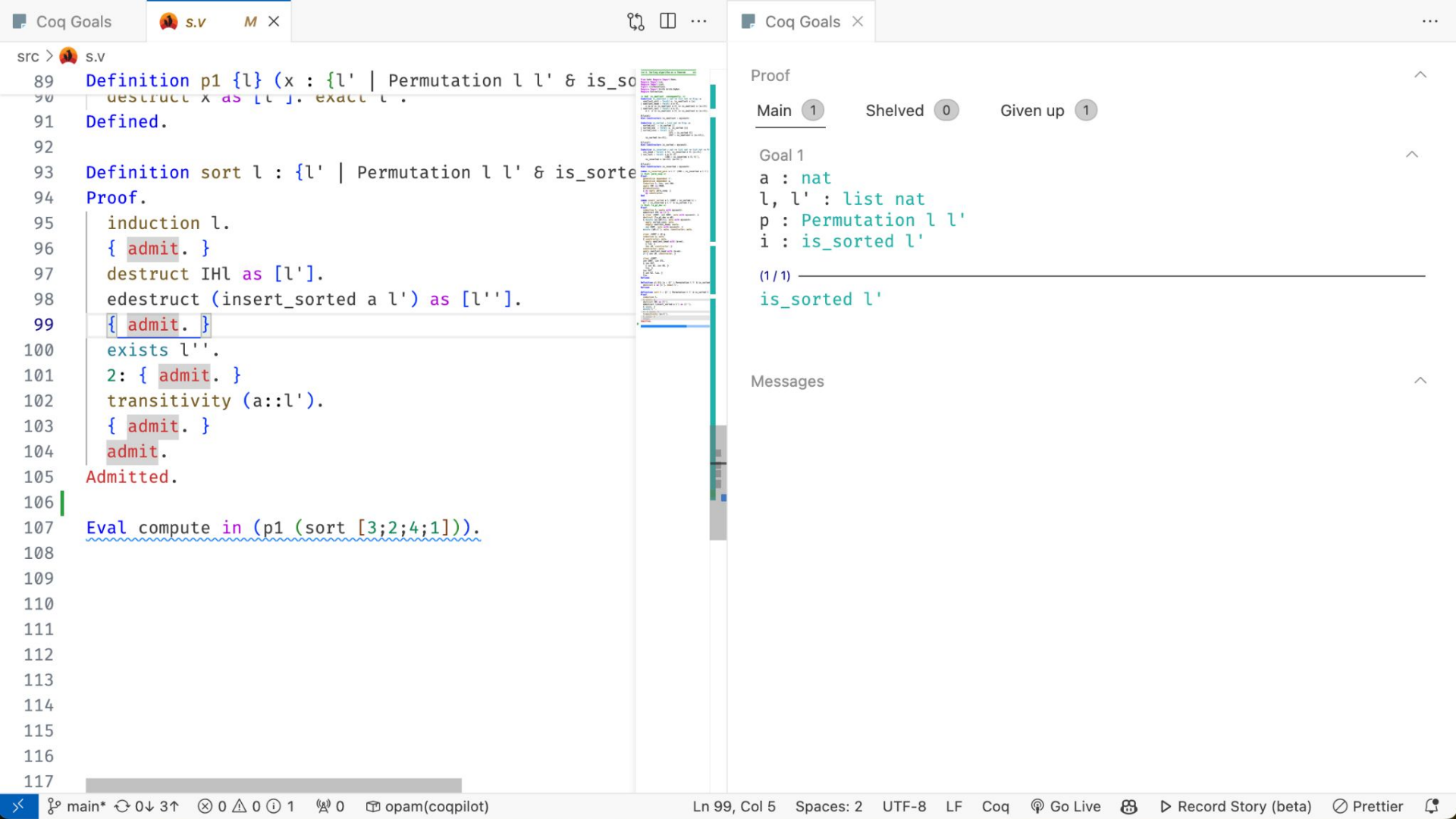
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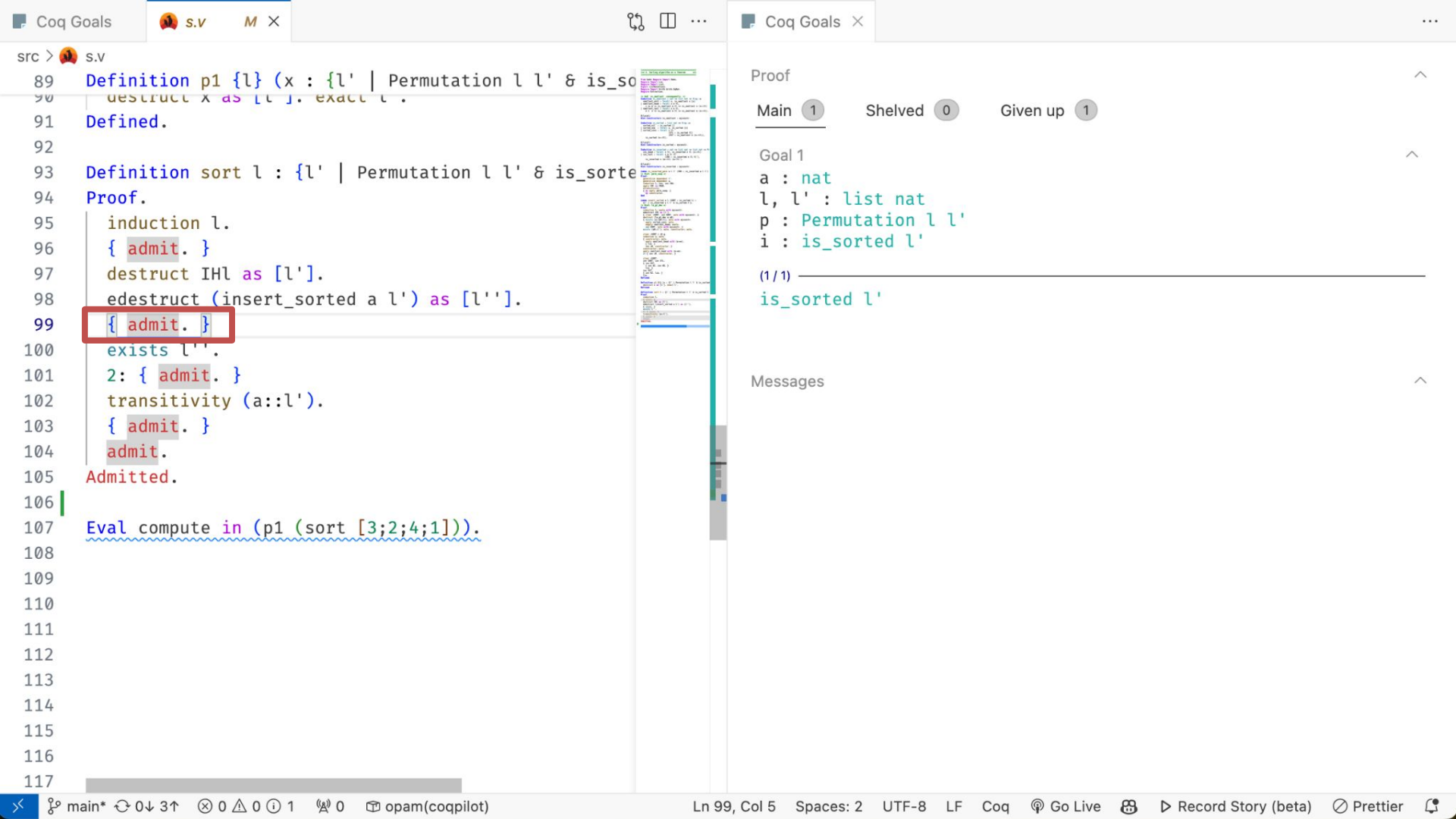
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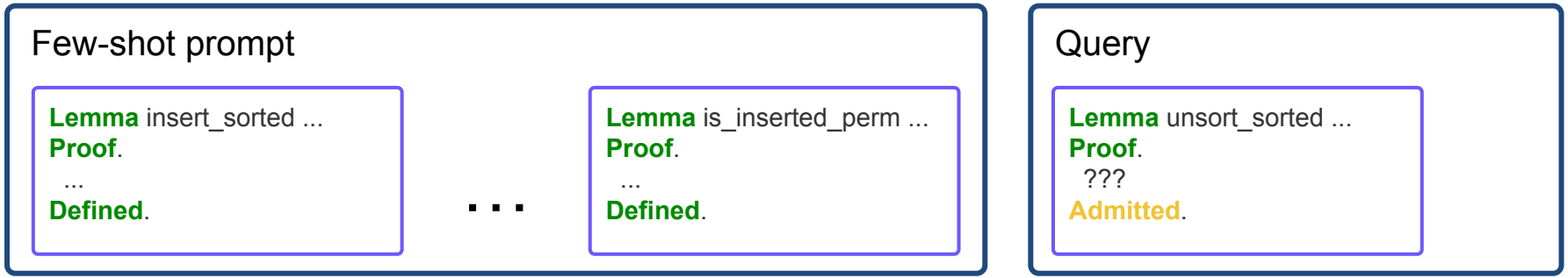
Few-shot prompt
Lemma insert_sorted ...
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. . .
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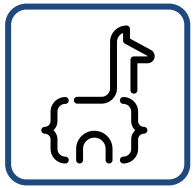
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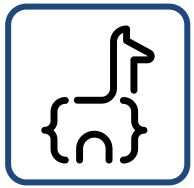
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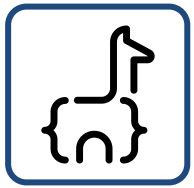
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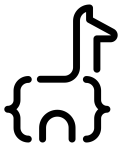
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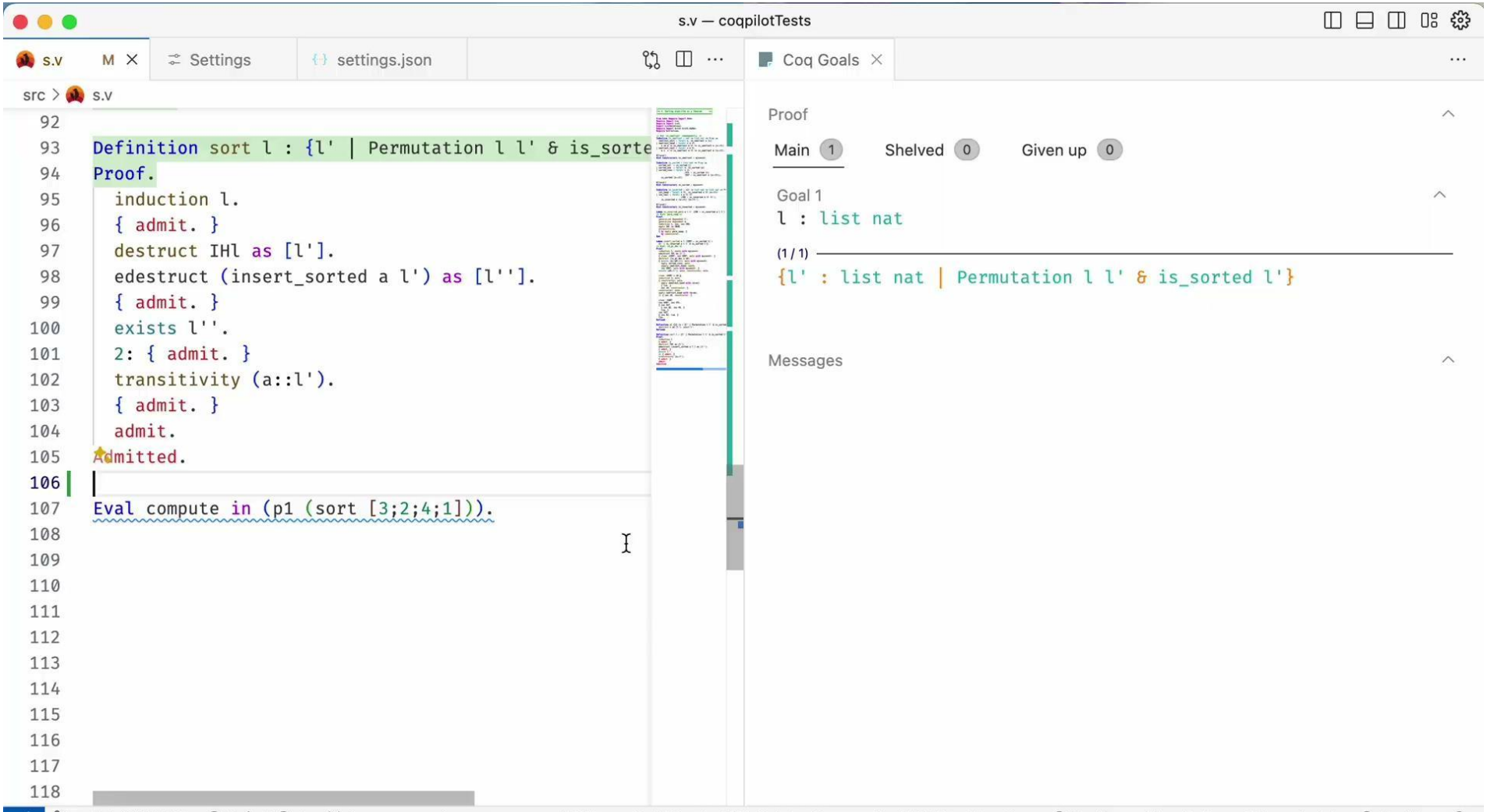
s.v — coqpilotTests
s.v M
Settings
settings.json
Coq Goals
src > s.v
92
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Proof repair

❌

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❌

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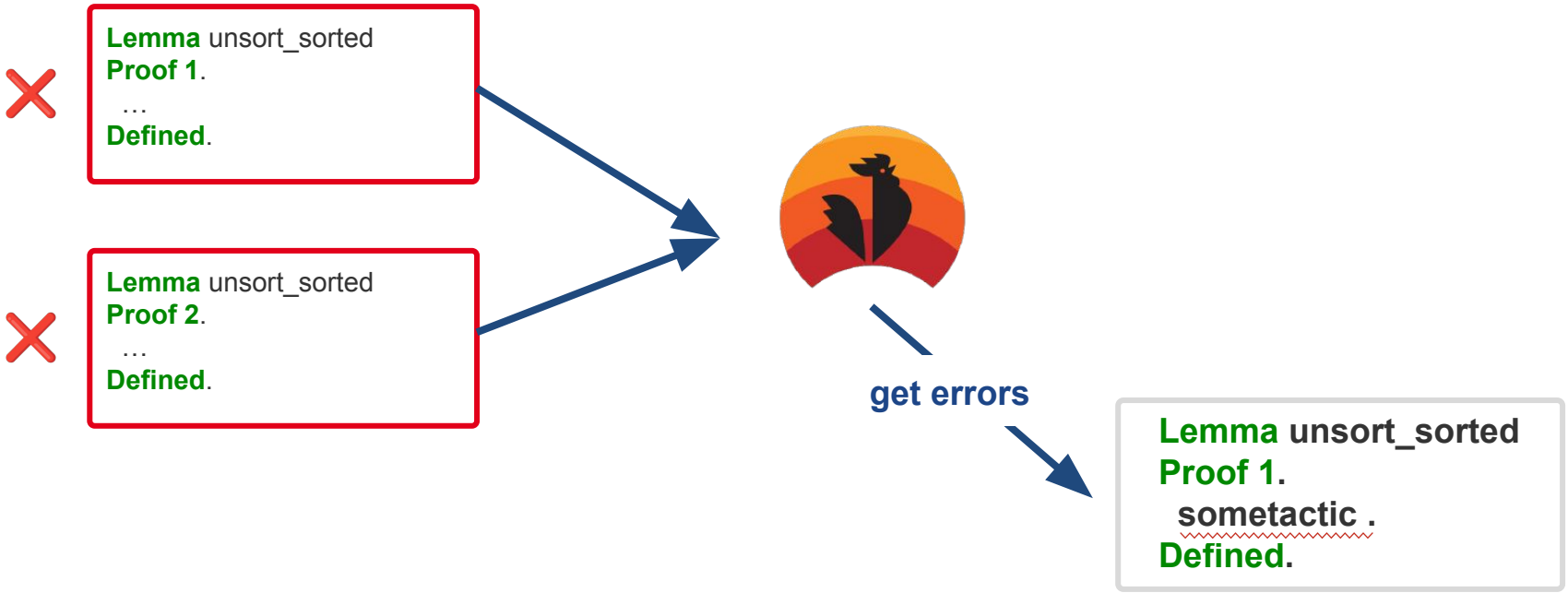
Proof repair



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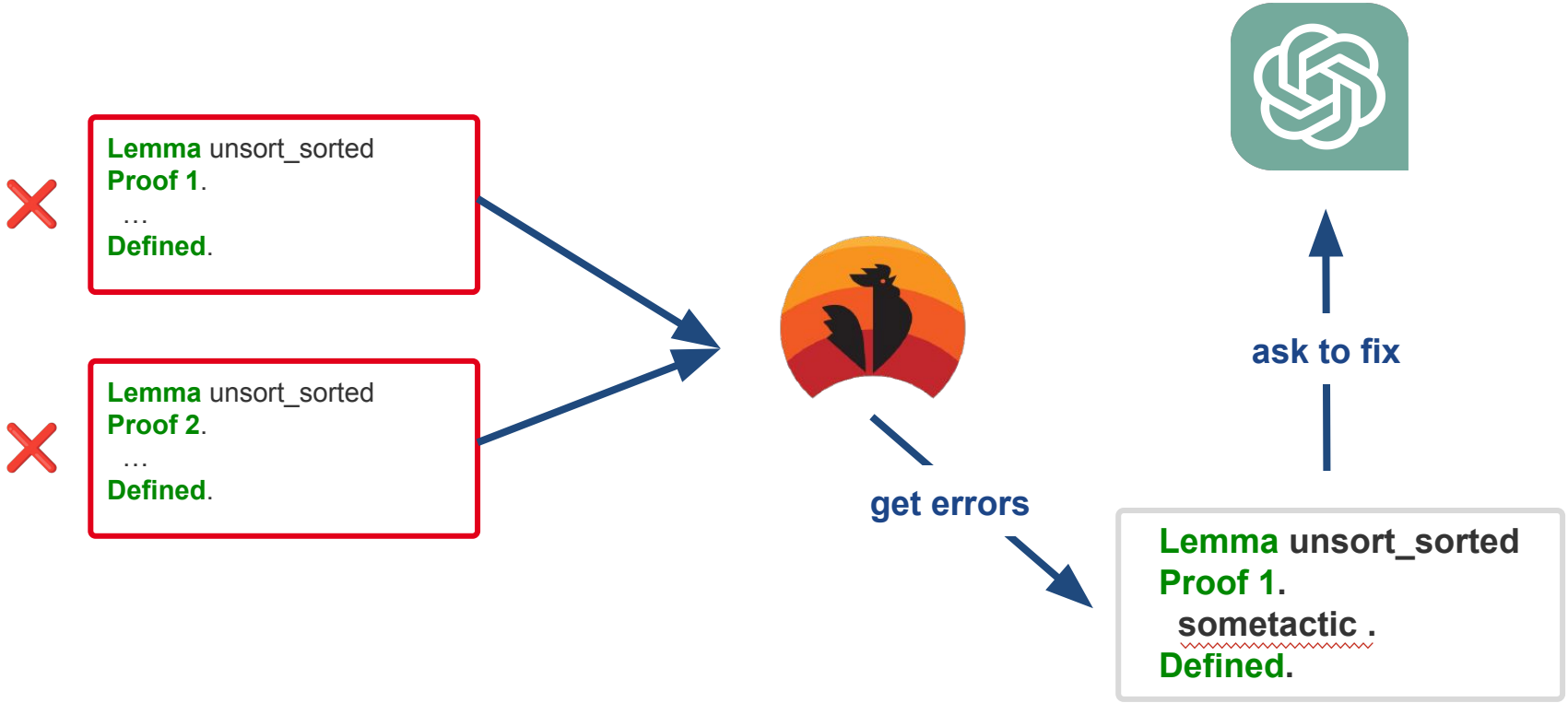

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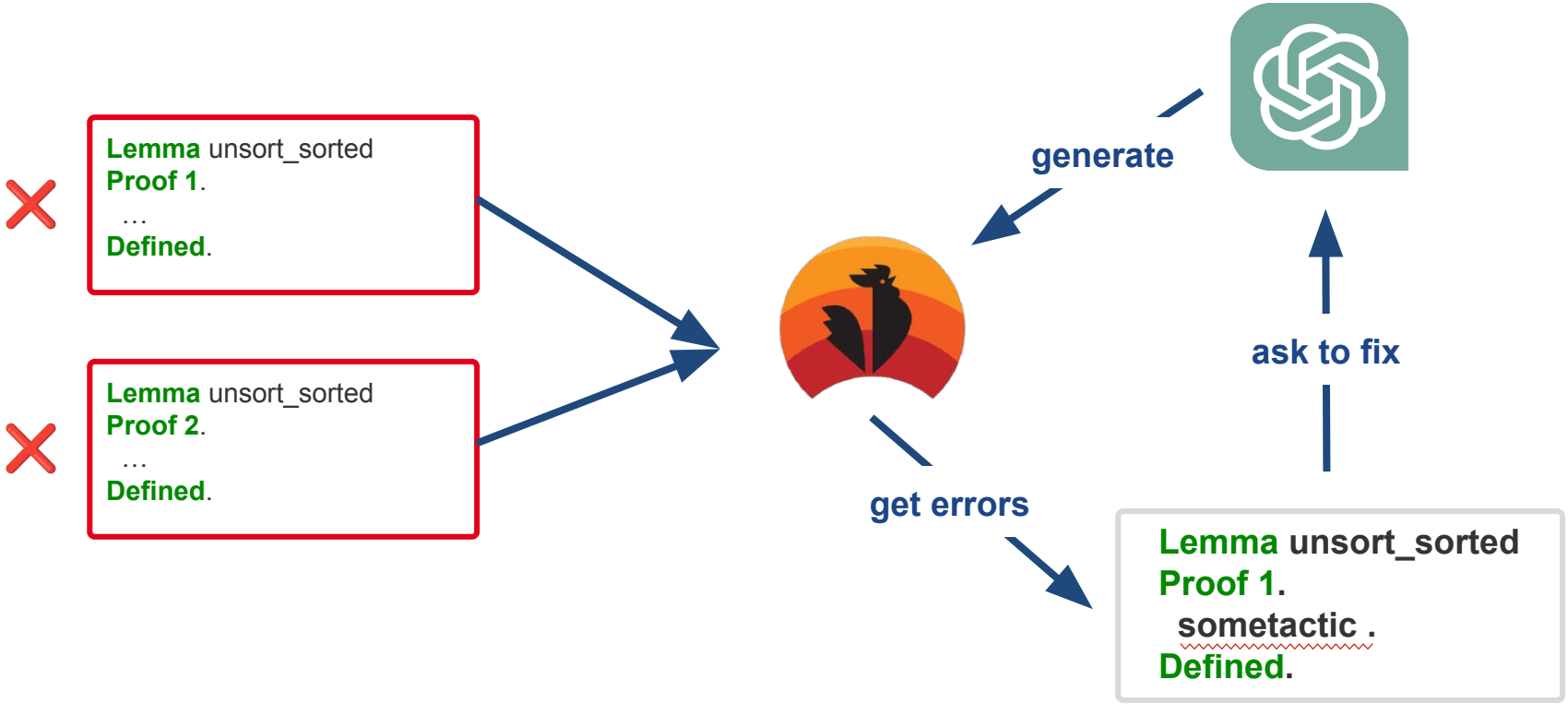

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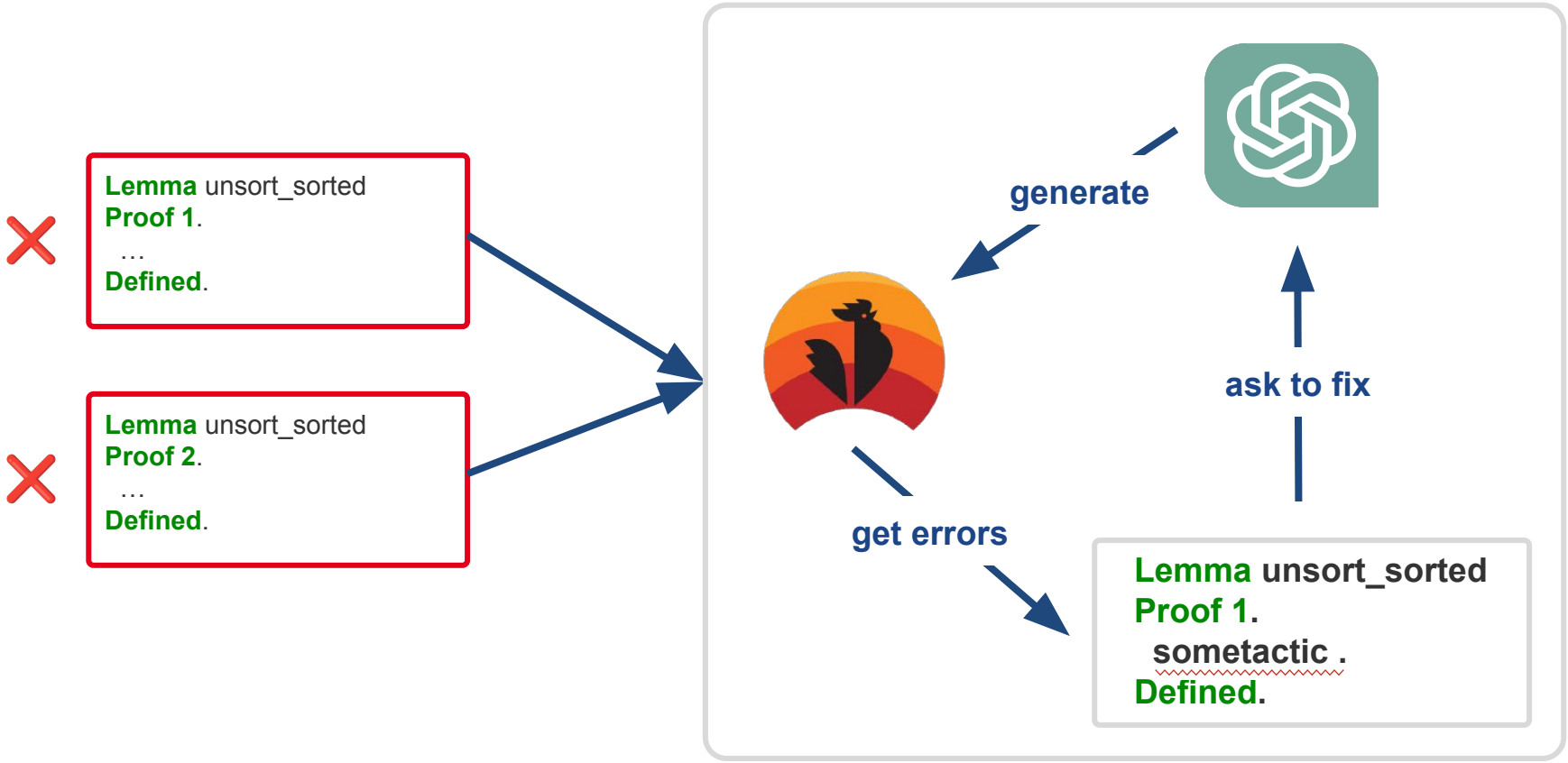

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- **RQ2**: To which extent does CoqPilot improve the LLM approach to Coq generation?
- **RQ3**: What is the additional value CoqPilot contributes to other Coq automation tools such as CoqHammer and Tactician?

Benchmarking Framework

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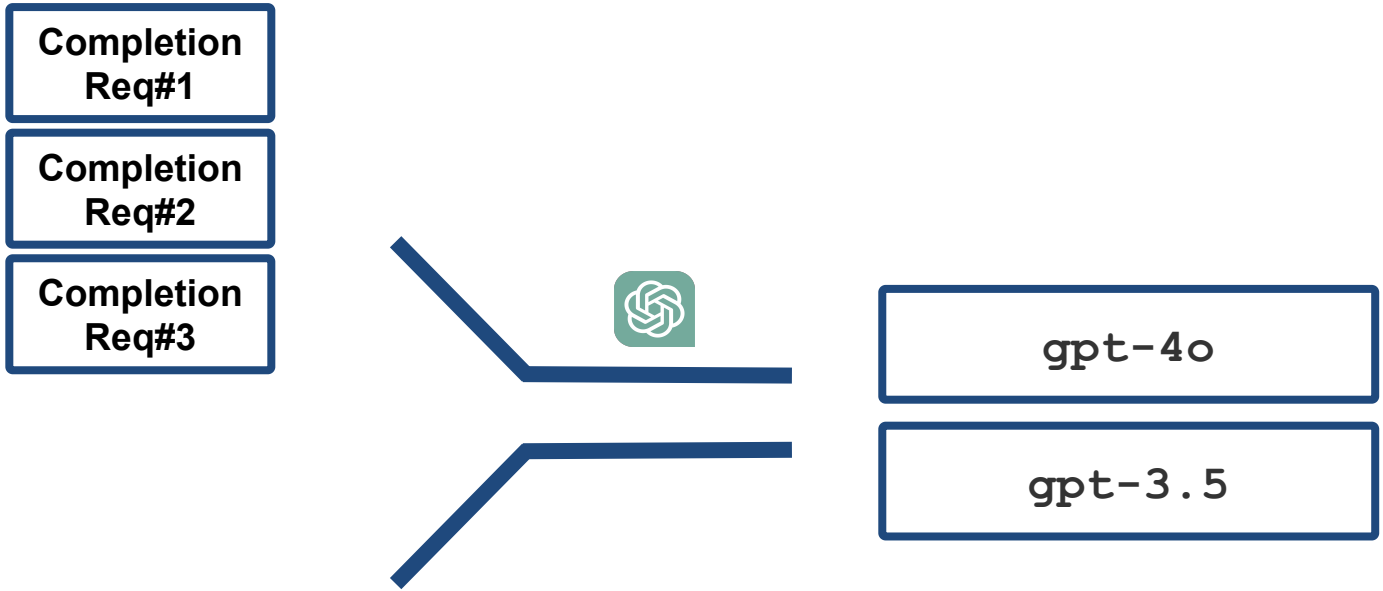

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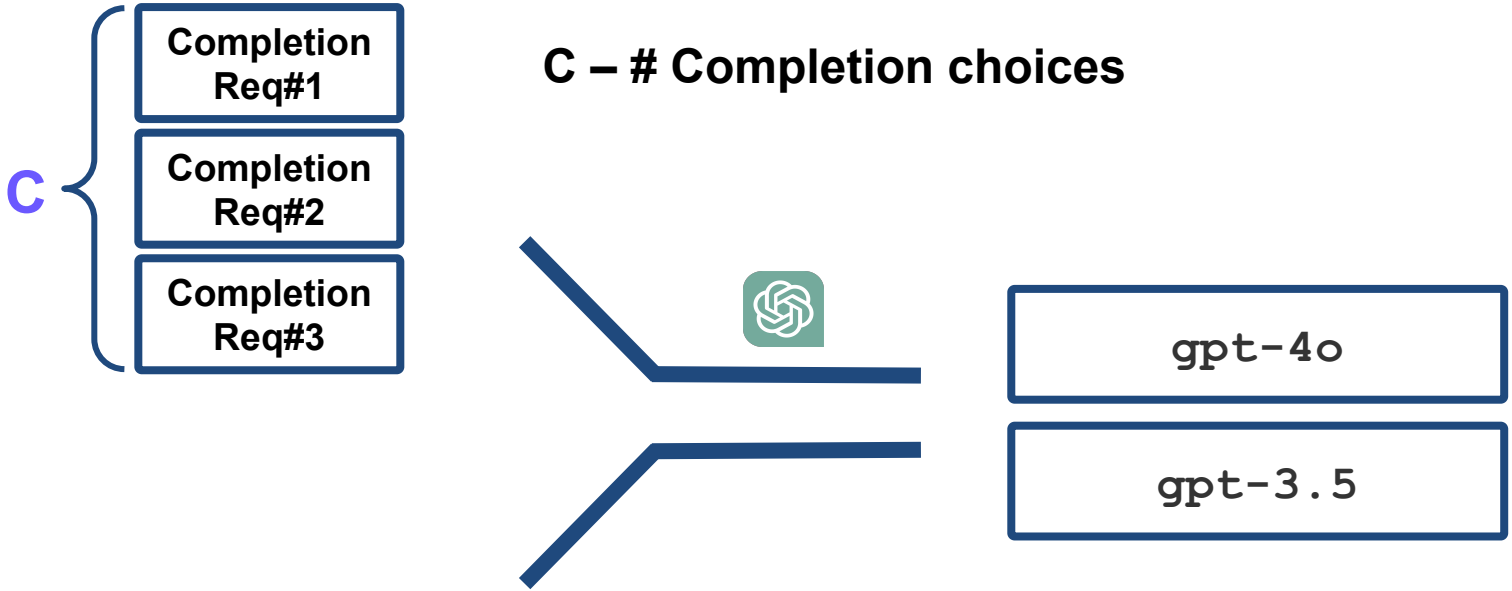

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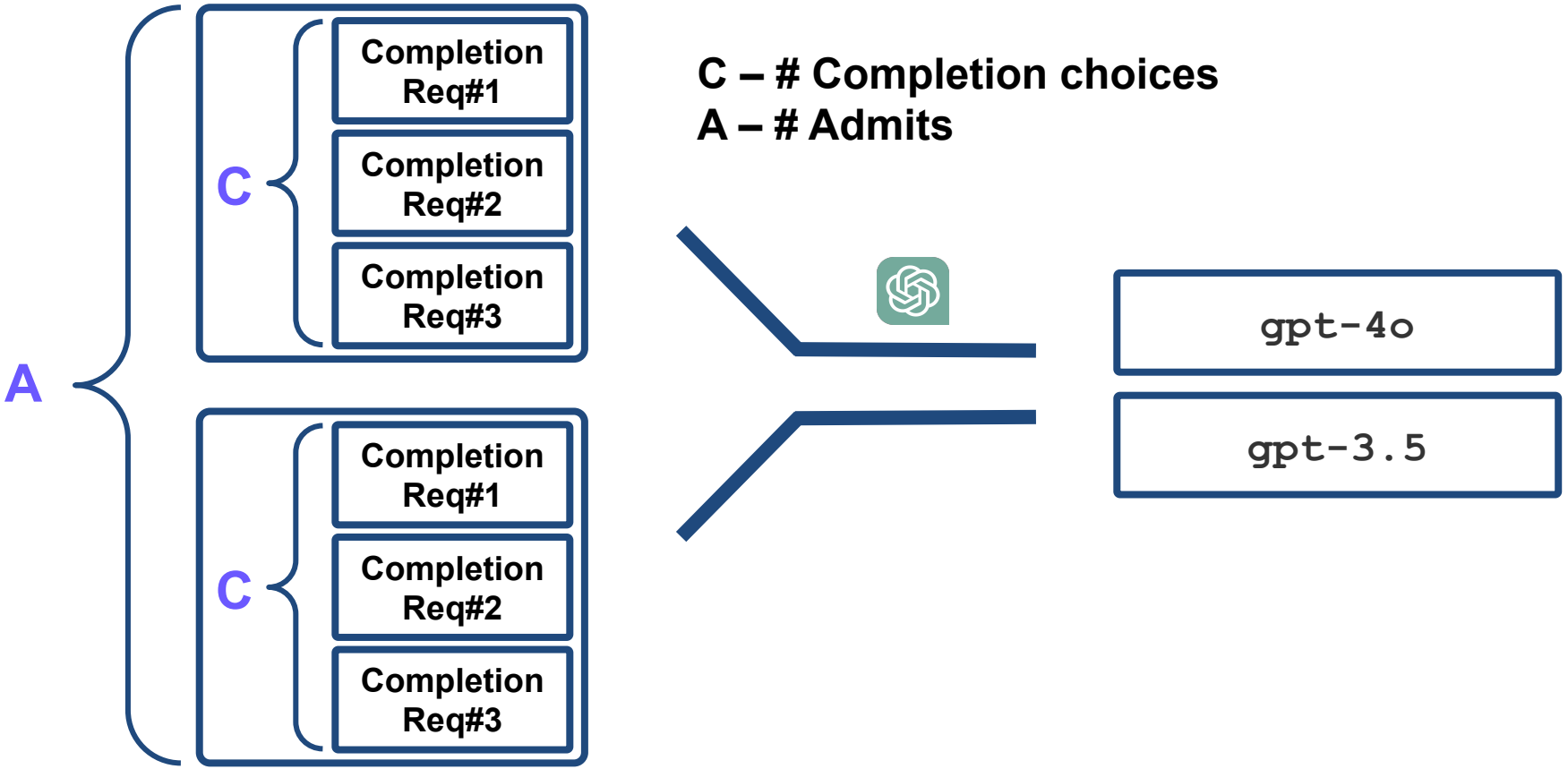
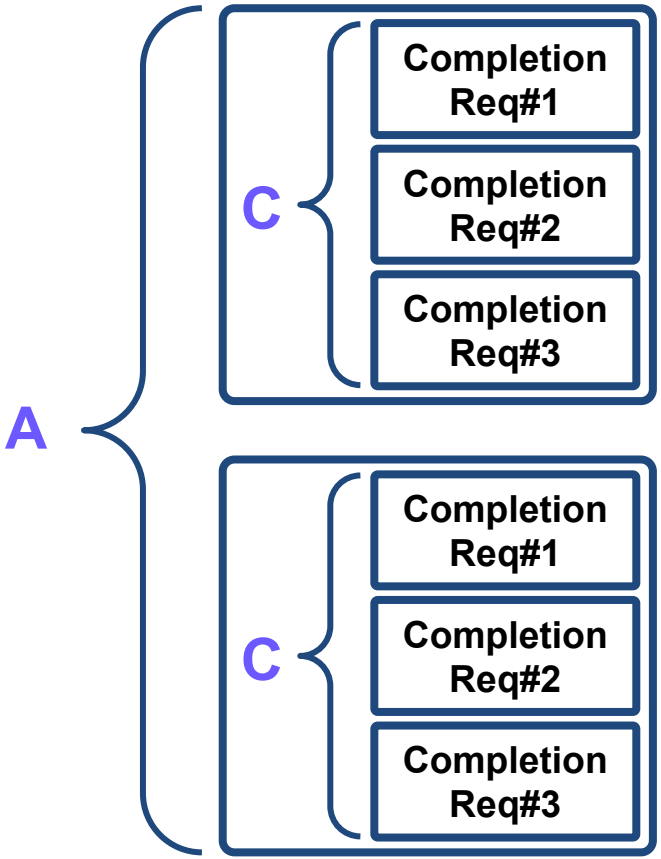
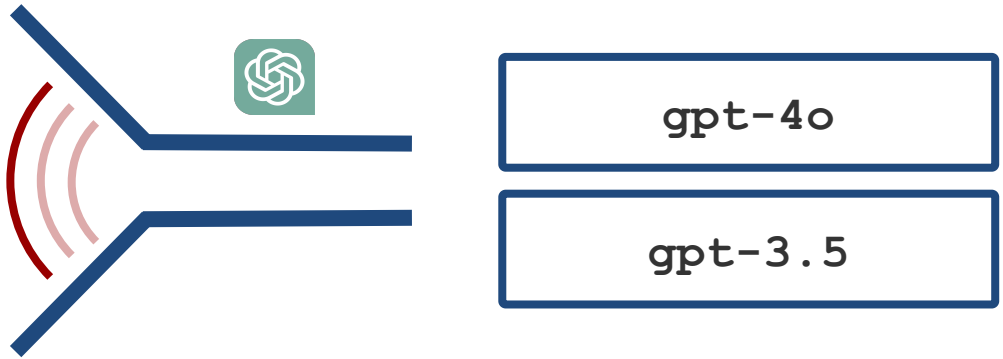


Benchmarking Framework

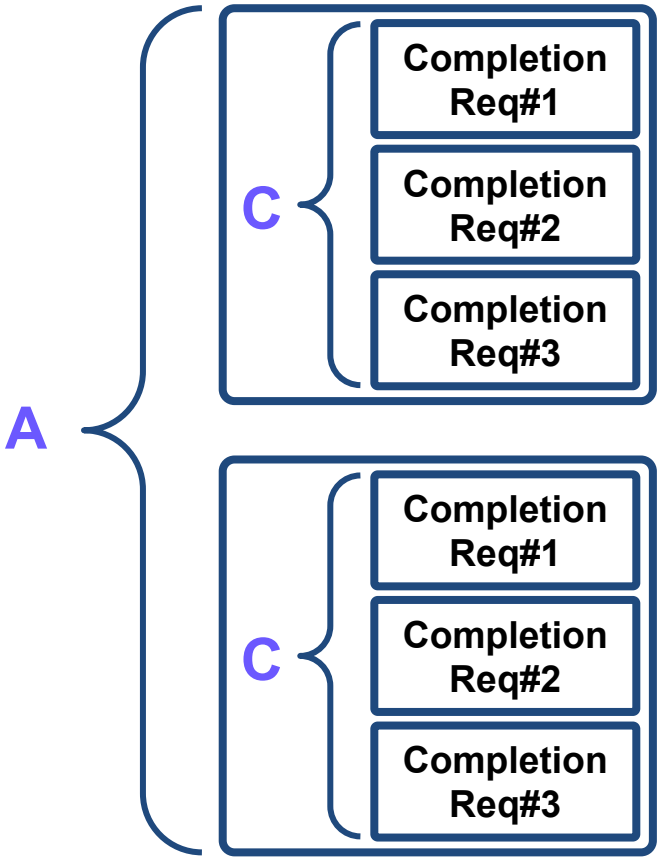


C – # Completion choices
A – # Admits



1. **Contextual limits**
2. Token rate limits
3. Request rate limits

Benchmarking Framework



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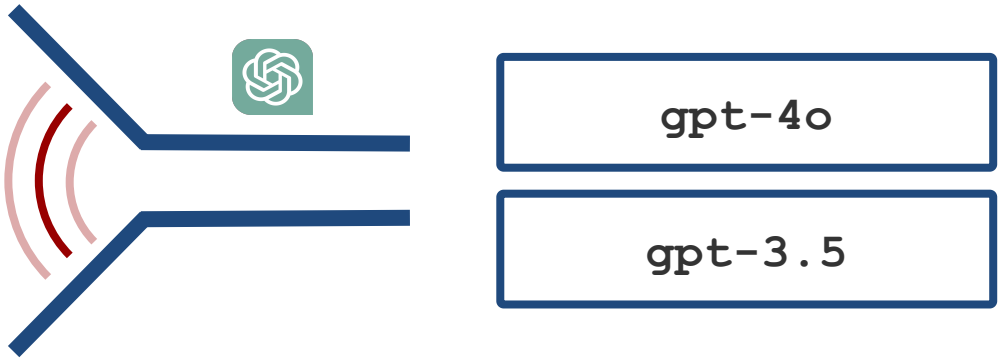

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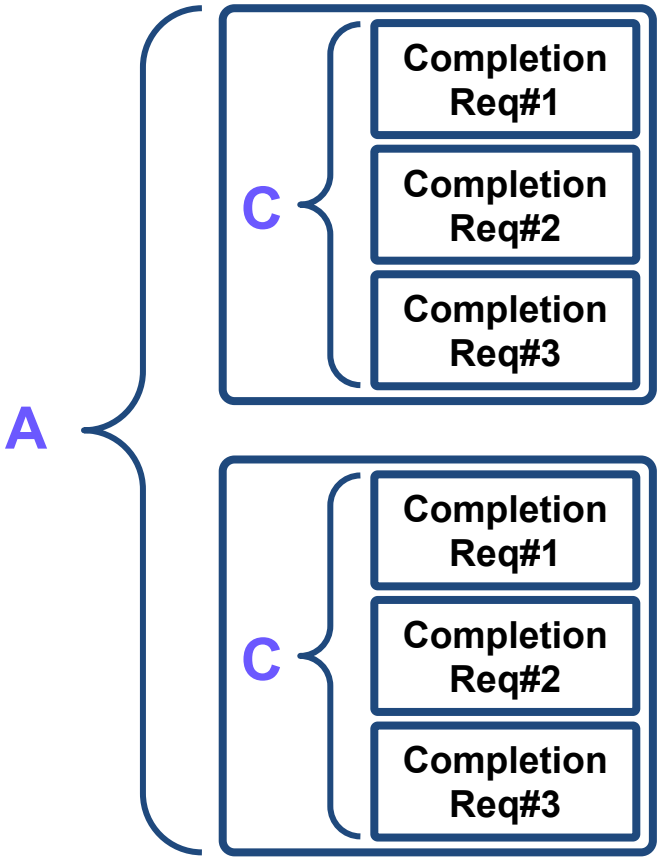

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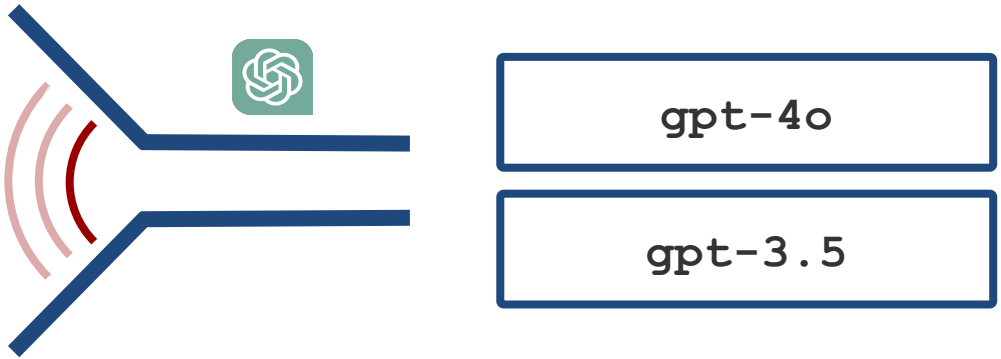

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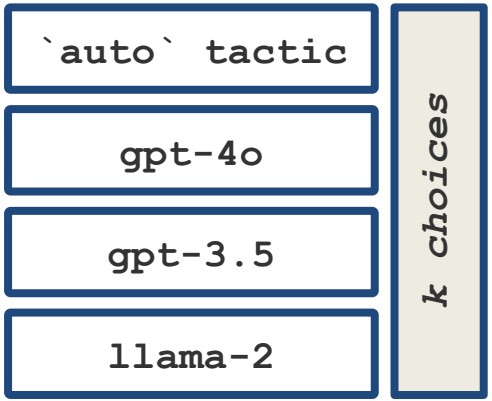

Results

In the table below you can find the results of our experiments. For each of the theorems we show its group and the methods which proved the theorem during our experiments.

File	Theorem Name	Predefined tactic	OpenAI GPT-4	OpenAI GPT-3.5	LLaMA-2 13B Chat
basic/Execution_eco.v	`rf_rmw_ct_in_co`	×	✓	×	×
imm/imm_hb.v	`cr_hb_hb`	×	✓	✓	×
basic/FinExecution.v	`fin_exec_same_events`	✓	✓	✓	×
basic/Execution.v	`sb_trans`	×	×	×	×
basic/Execution.v	`sb_same_loc_W_trans`	×	✓	×	×
basic/Events.v	`restr_eq_rel_same_loc`	✓	✓	✓	✓
basic/Events.v	`same_loc_loceq`	✓	✓	✓	×

Benchmarking Framework

```
interface Benchmark {
    name: string;
    items: DatasetItem[];
    inputModelsParams: InputModelsParams;
    requireAllAdmitsCompleted: Boolean;
    benchmarkFullTheorems: Boolean;
    benchmarkAdmits: Boolean;
    timeoutMinutes: number;
    groupName: string;
    additionalImports?: AdditionalFileImport[];
    maximumUsedPremisesAmount?: number;
    perProofTimeoutMillis: number;
}
```

Evaluation

Evaluation

Reference proof length	⩽ 4	5 – 8	9 – 20	Total
Group size	131	98	71	300
`firstorder auto with *`	11%	2%	1%	6%
OpenAI GPT-3.5	29%	17%	6%	20%
OpenAI GPT-4o	50%	26%	15%	34%
LLaMA-2 13B Chat	2%	0%	0%	0.5%
Anthropic Claude	21%	7%	7%	13%
All models together	57%	32%	18%	39%
Tactician	45%	23%	10%	29%
CoqHammer	23%	4%	0%	11%
All methods together	71%	45%	23%	51%

IMM Project: https://github.com/weakmemory/imm
CoqPilot ASE'24 Tool Demo paper on: https://podkopaev.net/

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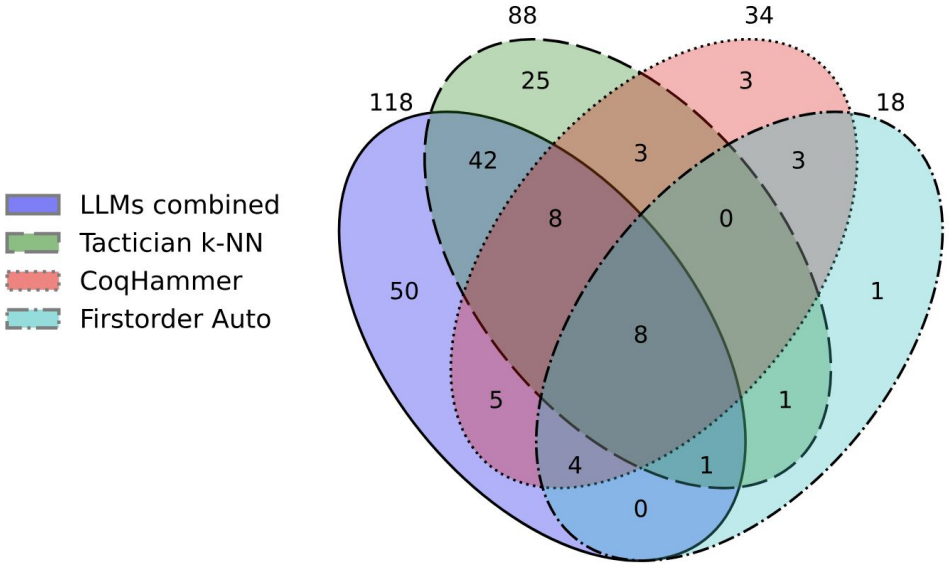

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Technical challenges: Coq-LSP

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Theorem plus : forall n:nat, 1 + n = S n.
Proof.
   admit.
Admitted.
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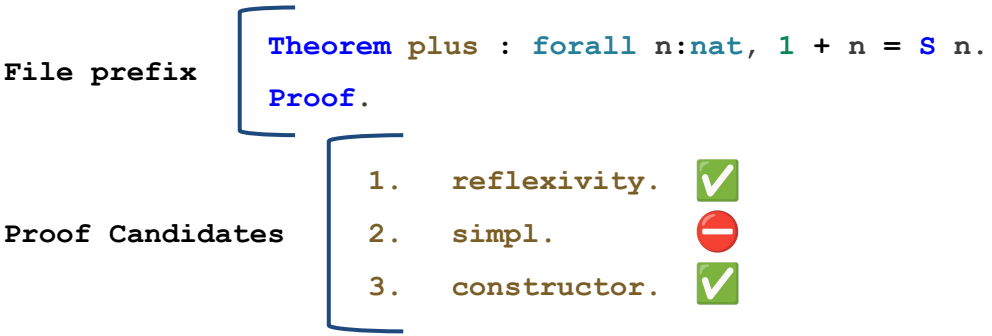

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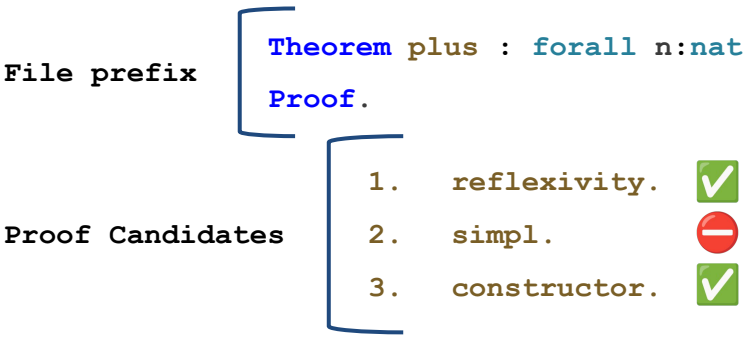


```
createTempFile()
file.write(prefix)
lsp.openDocument()
for proof in proofCandidates:
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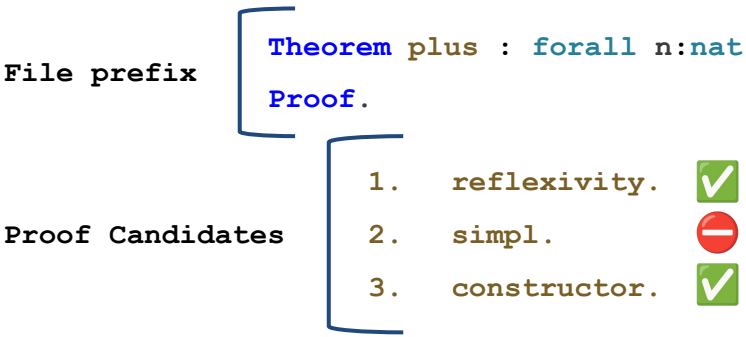


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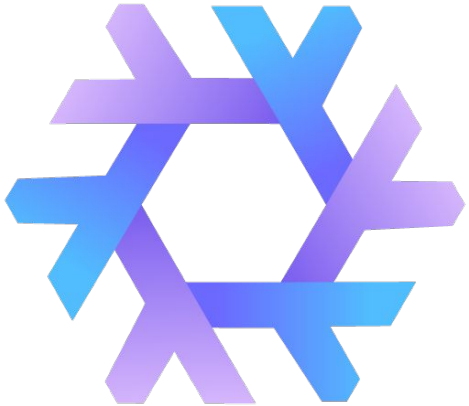
Technical challenges: Nix (1/2)



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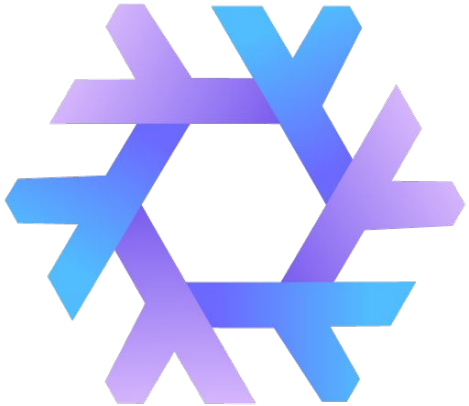
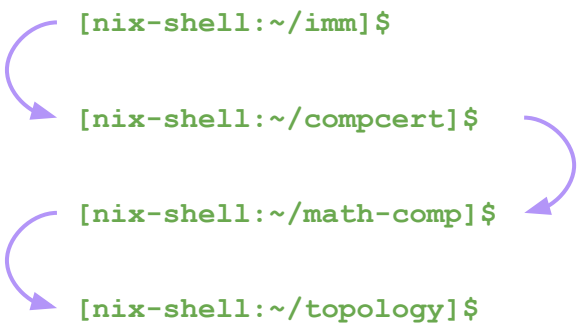
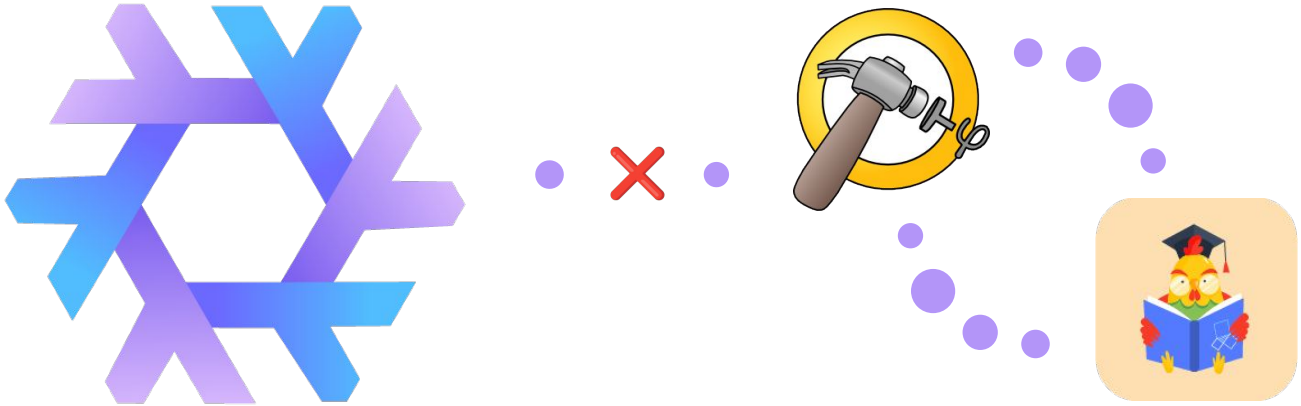




Technical challenges: Nix (2/2)



Improvement directions

Improvement directions

Integrate more Coq generation tools, such as Copra, Graph2Tac, TacTok etc.

Improvement directions

Improve premise selection and add new selection techniques

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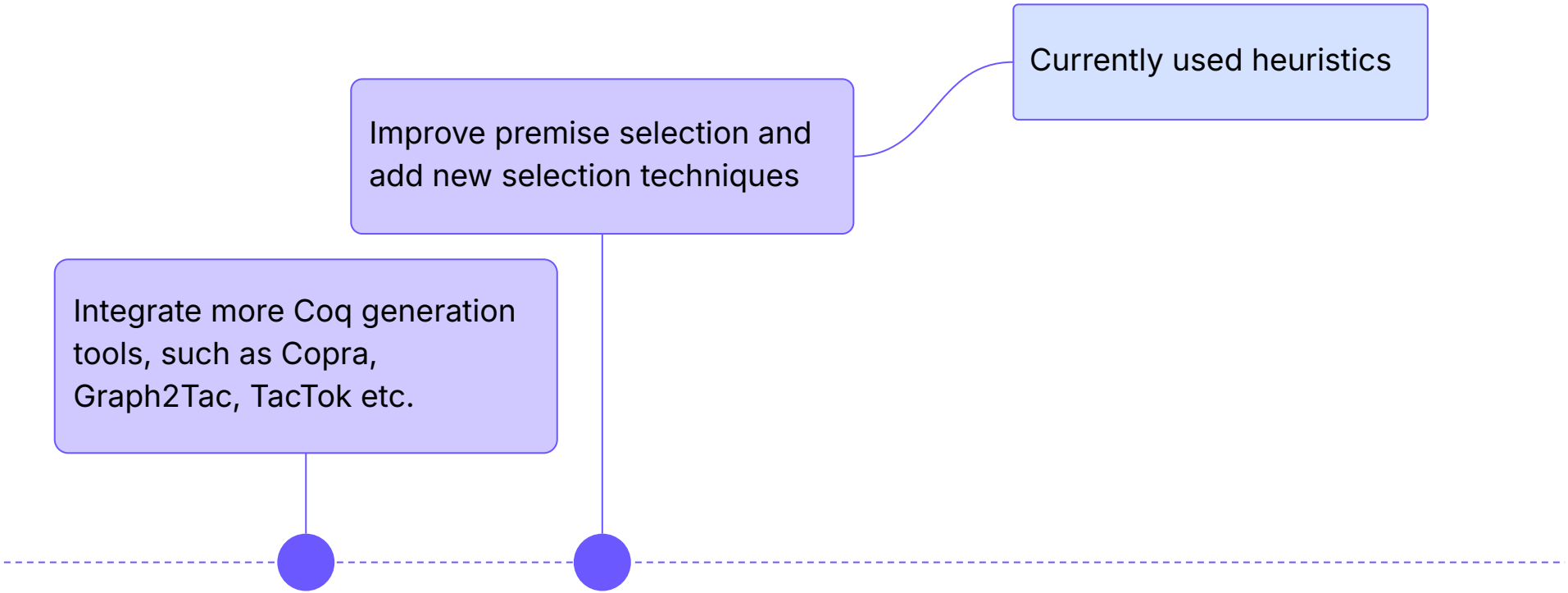

Improvement directions

Improve premise selection and add new selection techniques

Currently used heuristics

Distance from the target theorem

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Improve premise selection and add new selection techniques

Currently used heuristics

Distance from the target theorem

Similarity of the goal

Improvement directions

Improve premise selection and add new selection techniques

Integrate more Coq generation tools, such as Copra, Graph2Tac, TacTok etc.

Explore and improve locally available models in order to make inference cheaper and preserve privacy

Improvement directions

Improve premise selection and add new selection techniques

Integrate more Coq generation tools, such as Copra, Graph2Tac, TacTok etc.

Please talk to us if you have ideas!

CoqPilot: a plugin for LLM-based generation of proofs

JetBrains-Research/**coqpilot**

extension: **coqpilot**

{andrei.kozyrev, gleb.solovev, nikita.khramov, anton.podkopaev}@jetbrains.com

Programming Languages and Program Analysis Lab (PLAN), JetBrains Research