Coq workshop 2021

SSProve

Modular Cryptographic Proofs in Coq

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Cătălin **Hriţcu** Kenji **Maillard** Bas **Spitters**

2004

“Crisis of rigour” in crypto

Shoup, Bellare, Rogaway

Game playing proofs

2004

“Crisis of rigour” in crypto

Shoup, Bellare, Rogaway

Game playing proofs

A lot of good things

and more...

CertiCrypt

FCF

A lot of good things

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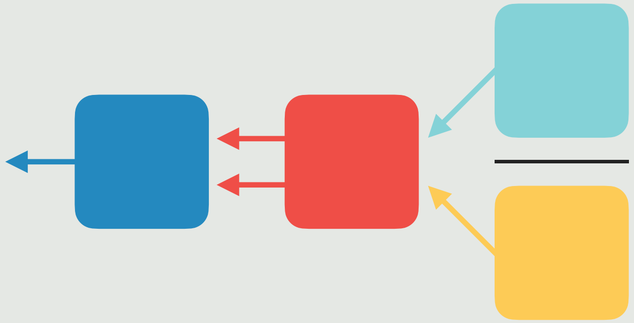
2018

State separating proofs SSP

Brzuska, Delignat-Lavaud, Fournet, Kohbrok, Kohlweiss

Modular/scalable high-level game playing proofs



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Modular/scalable high-level game playing proofs

2021

SSProve

CSF 2021

Semantics for SSP + relational program logic

SSP + 

2021

SSProve

CSF 2021

Semantics for SSP + relational program logic

SSP methodology

SSP methodology

Real

```
ENC(msg) :=
  if key = None then
    key ← sample 𝒟
  (r, c) ← enc(key, msg)
  return (r, c)
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SSP methodology

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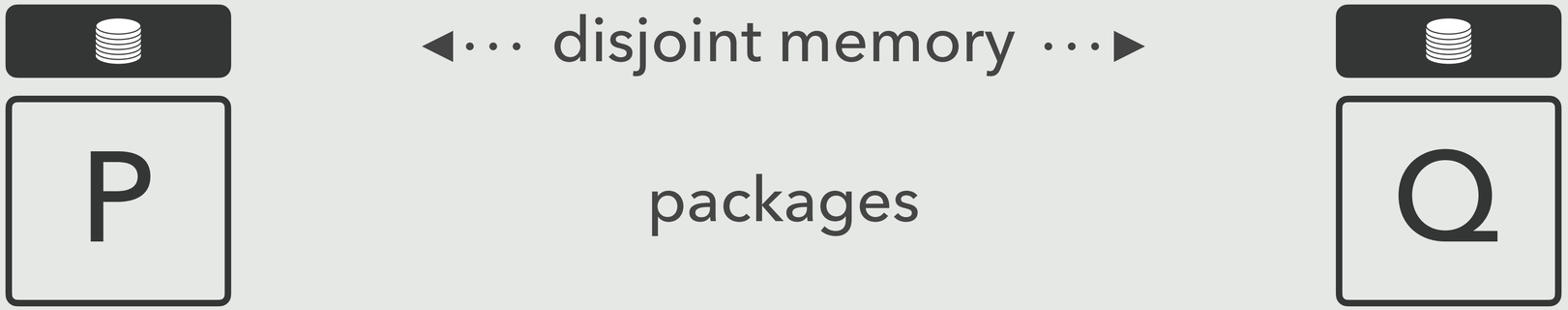

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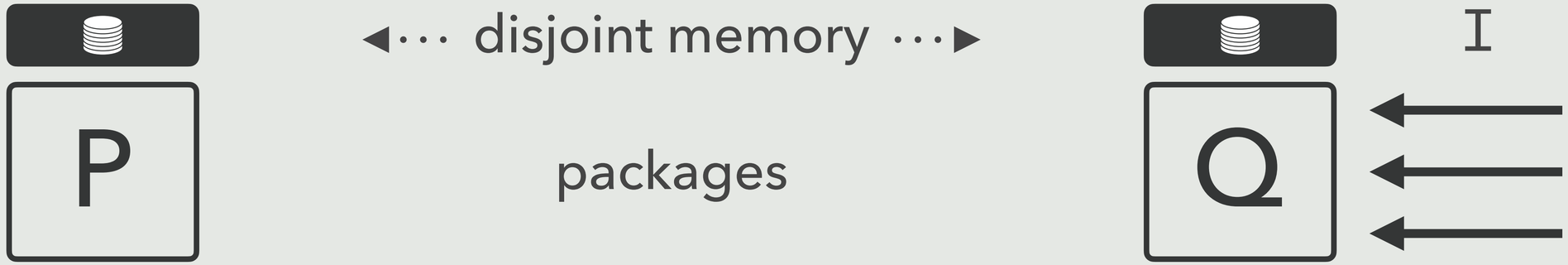

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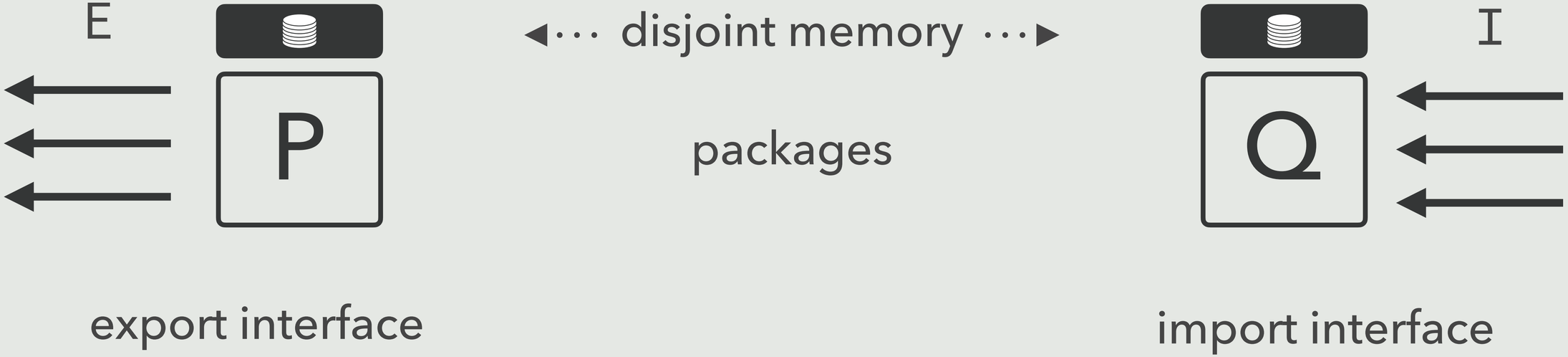

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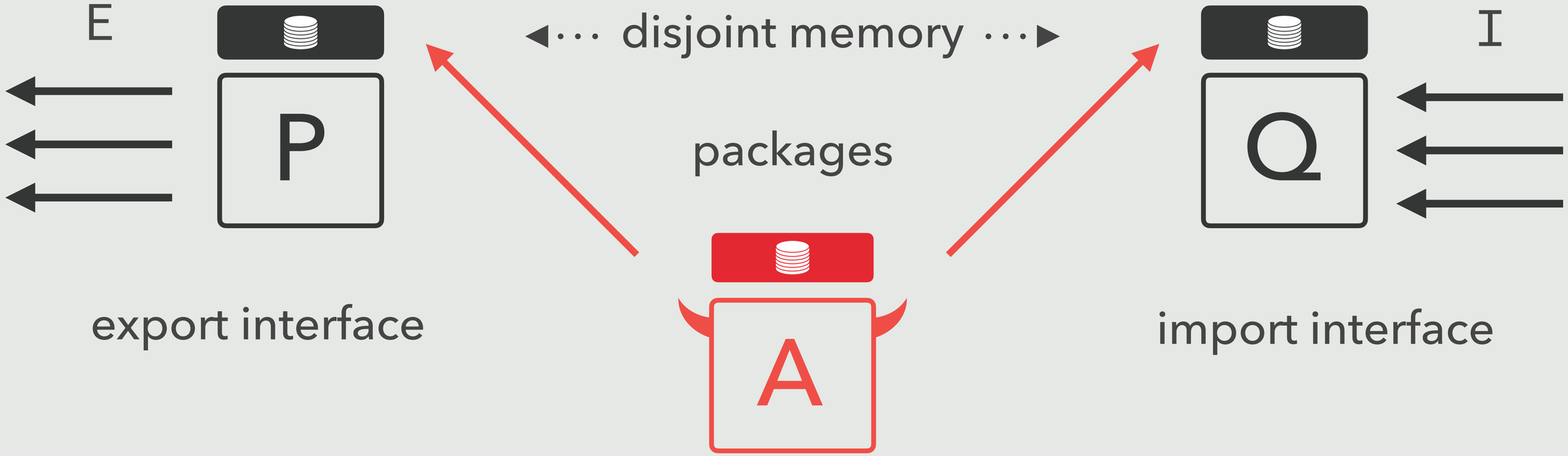

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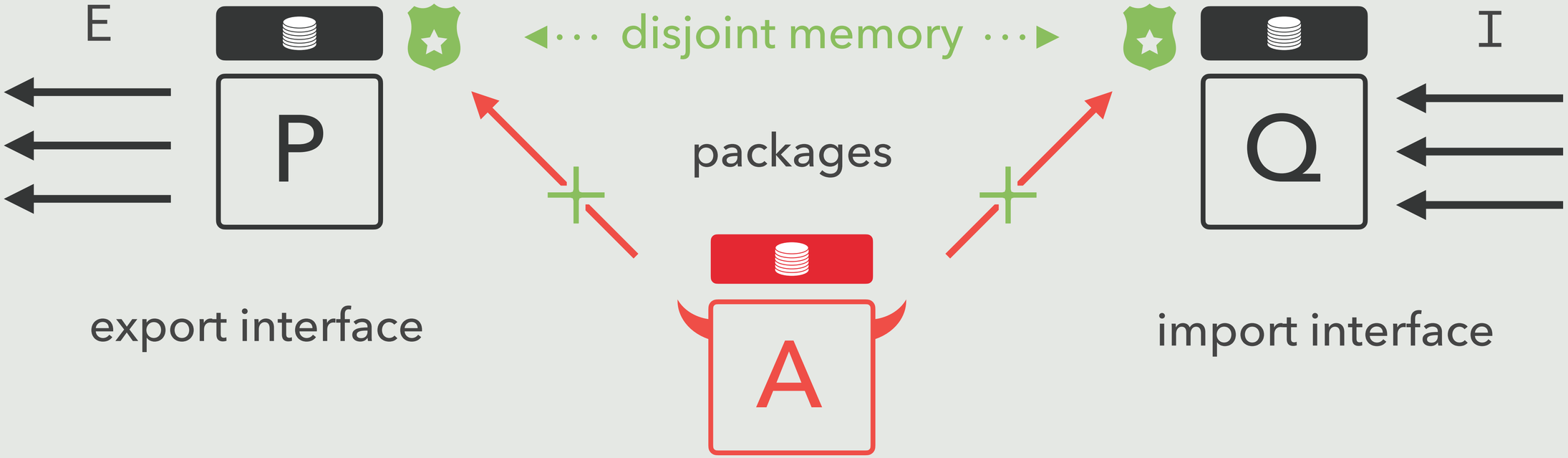

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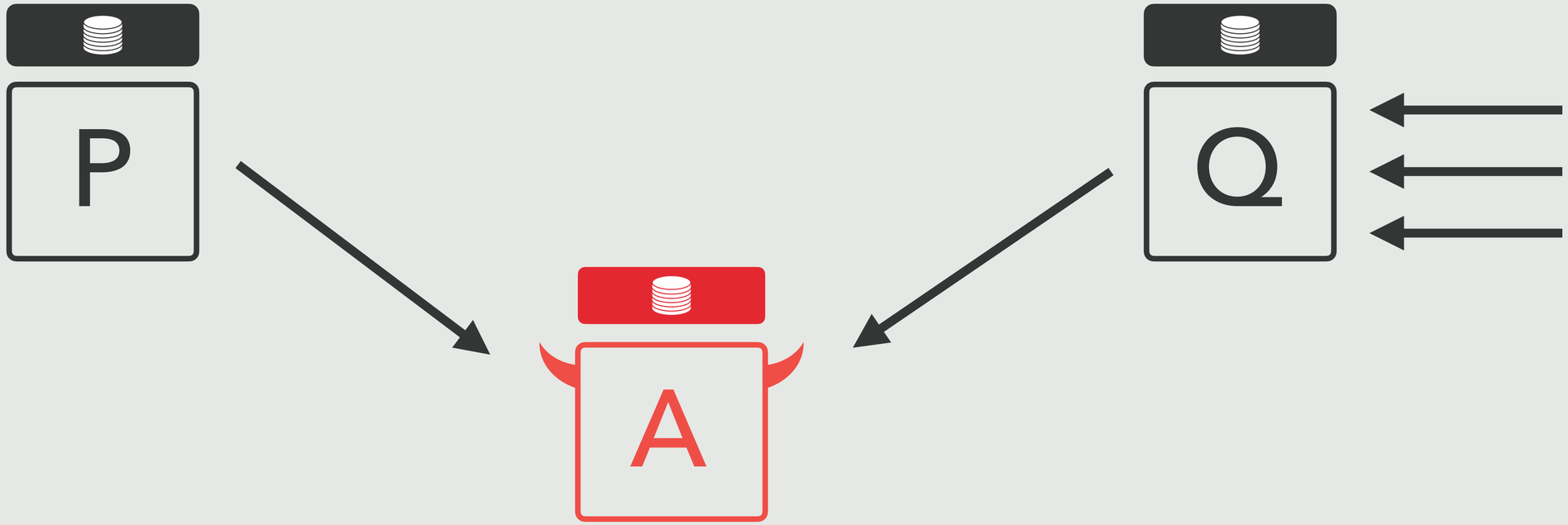

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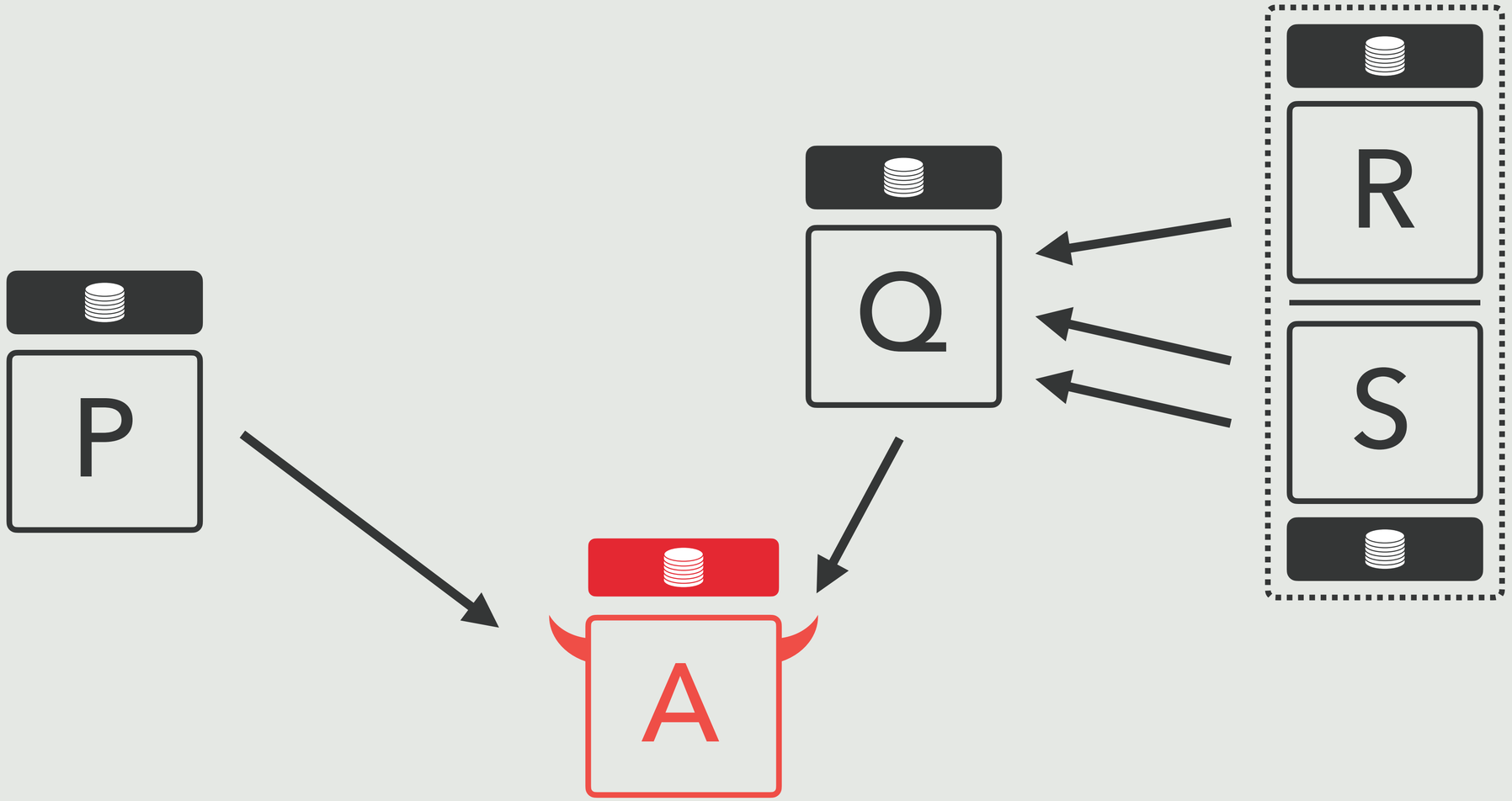
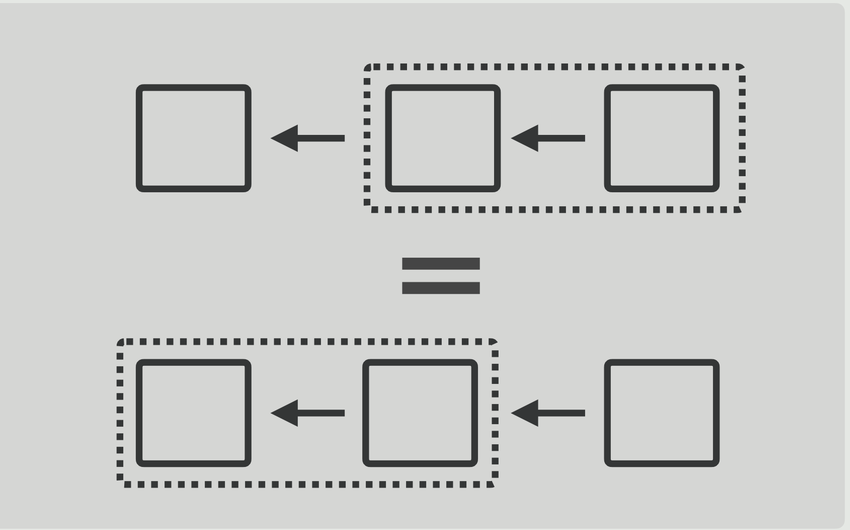
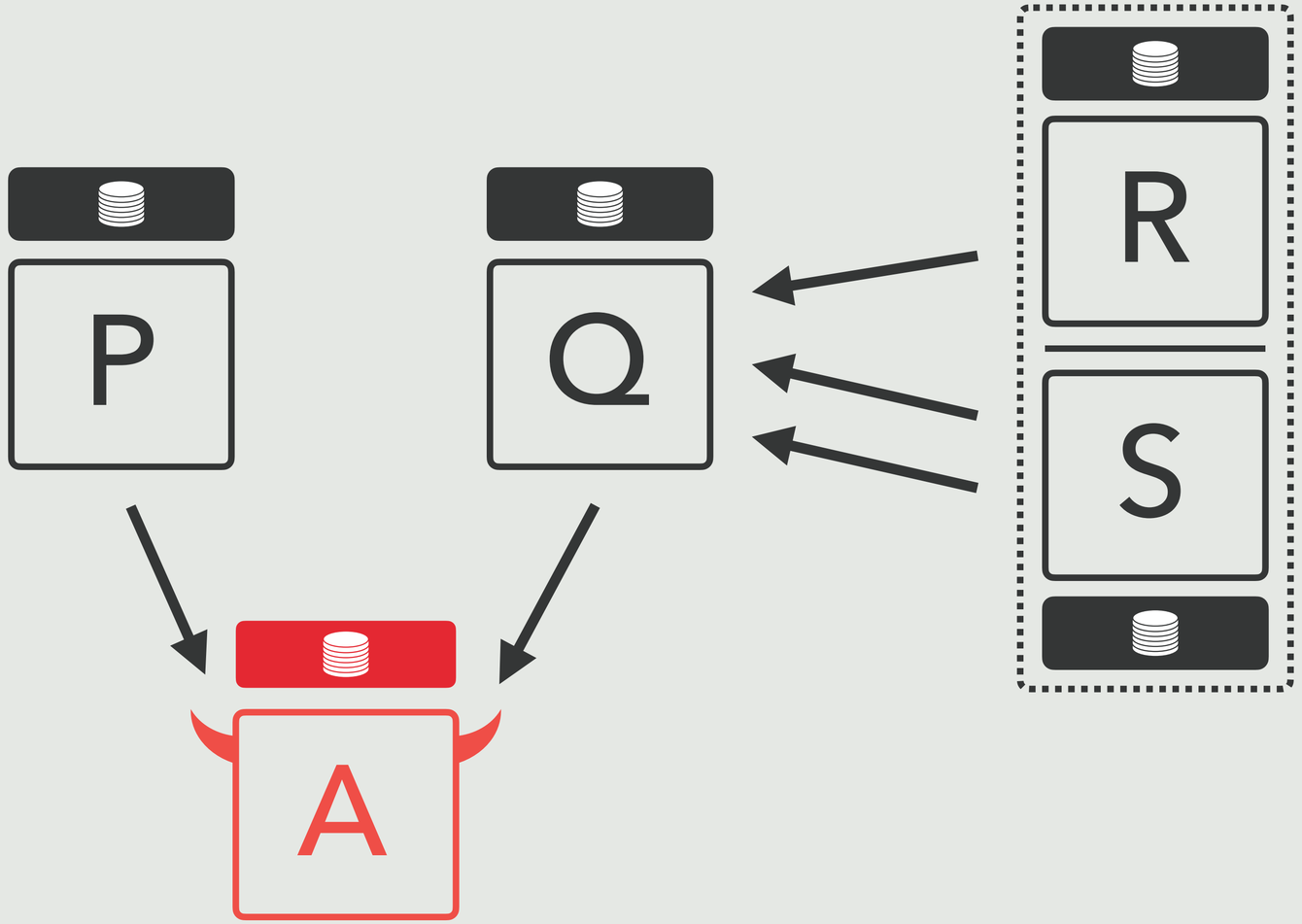
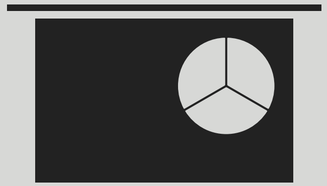


SSP methodology

package laws







More details

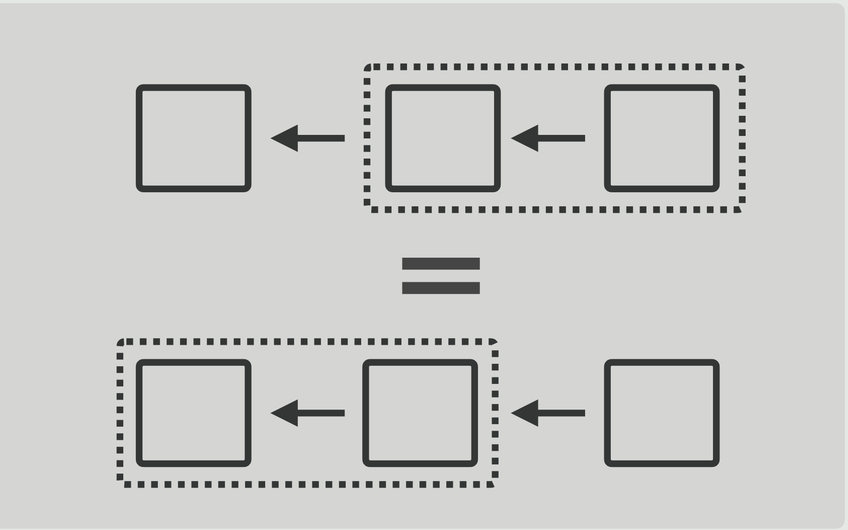
Philipp's talk at CSF 2021

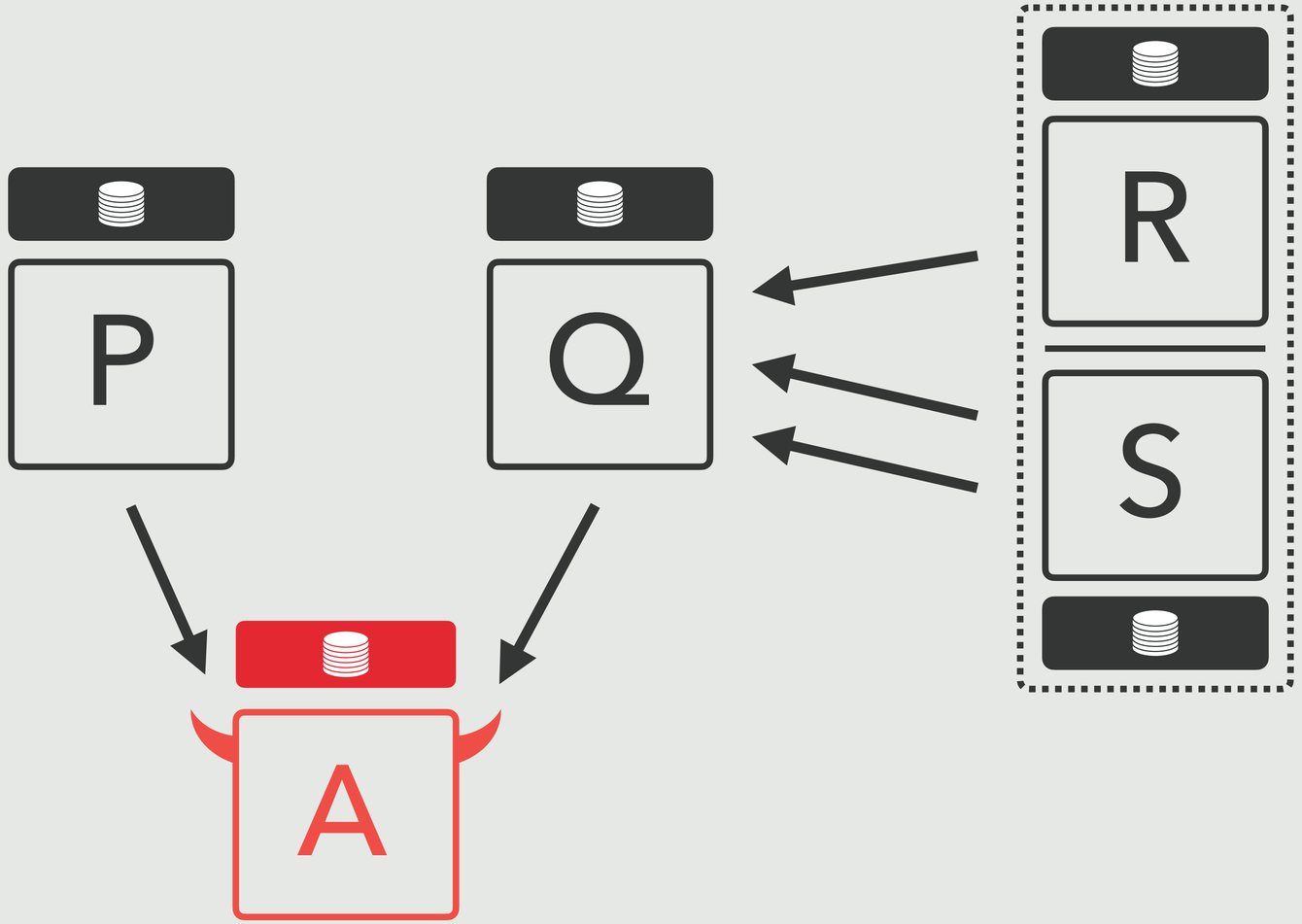
+

SSP paper

SSProve paper

package laws

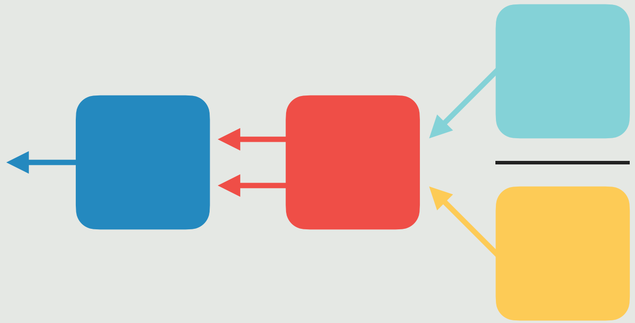




SSProve

formally defines

packages



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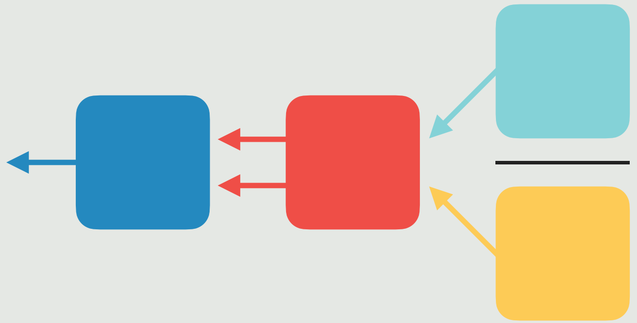

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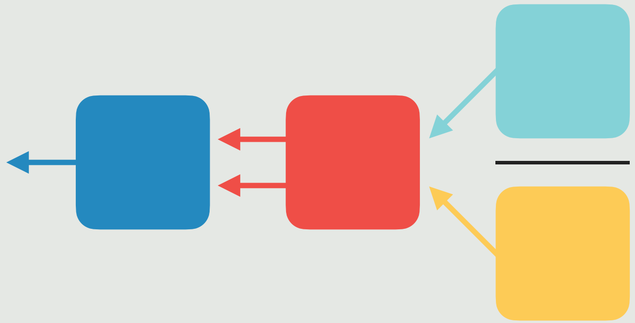

package laws

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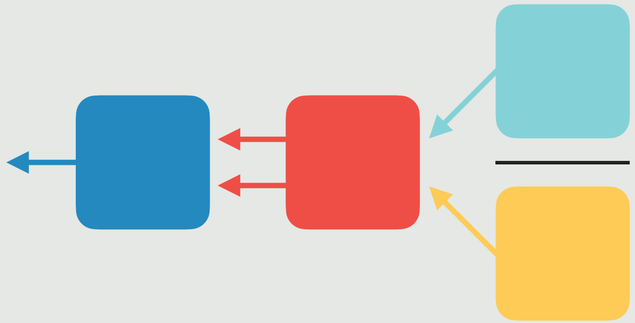
semantics

packages



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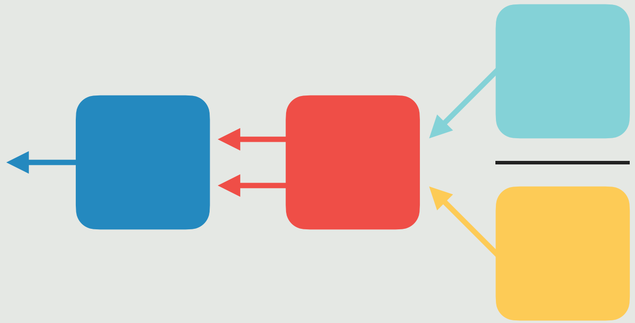

package laws

security notion

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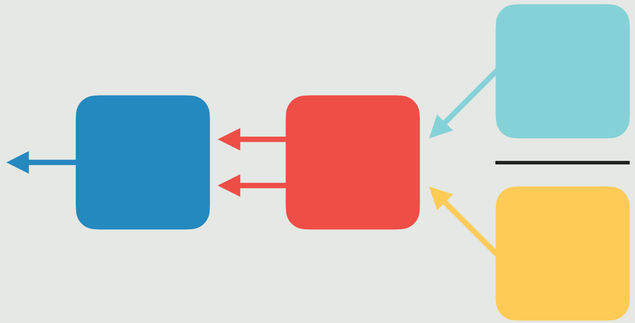

package laws

security notion

```
Inductive raw_code A : Type :=
| ret (x : A)
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Inductive raw_code A : Type :=
| ret (x : A)
| opr (o : opsig) (x : src o) (k : tgt o → raw_code A)
```

```
z ← op o · x ;; k z
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Inductive raw_code A : Type :=
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| getr (ℓ : Location) (k : ℓ → raw_code A)
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```
z ← op o · x ;; k z
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```
z ← get ℓ ;; k z
```

```
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```
put ℓ := v ;; k
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| putr (ℓ : Location) (v : ℓ) (k : raw_code A)
| sampler (𝒟 : Op) (k : Arit 𝒟 → raw_code A).
```

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z ← op o · x ;; k z
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```

```
#import {sig #[id] : 'nat → 'bool } as f ;; k f
```

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z ← get ℓ ;; k z
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put ℓ := v ;; k
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z ← sample 𝒟 ;; k z
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z ← get ℓ ;; k z
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put ℓ := v ;; k
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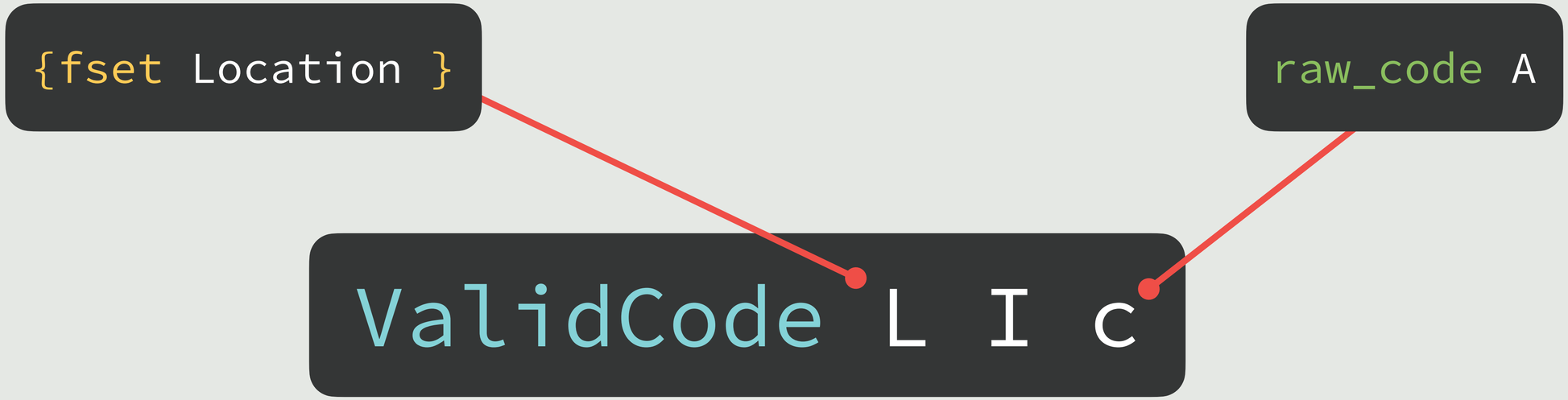
```
z ← sample 𝒟 ;; k z
```

```
z ← m ;; k z
```

raw_code A

ValidCode L I c


raw_code A
ValidCode L I c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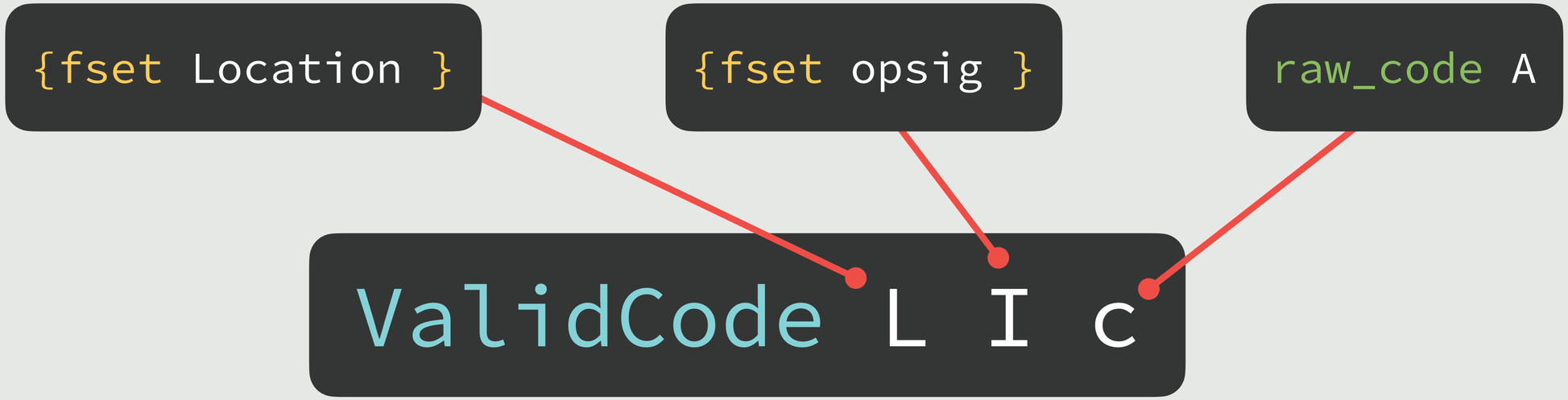
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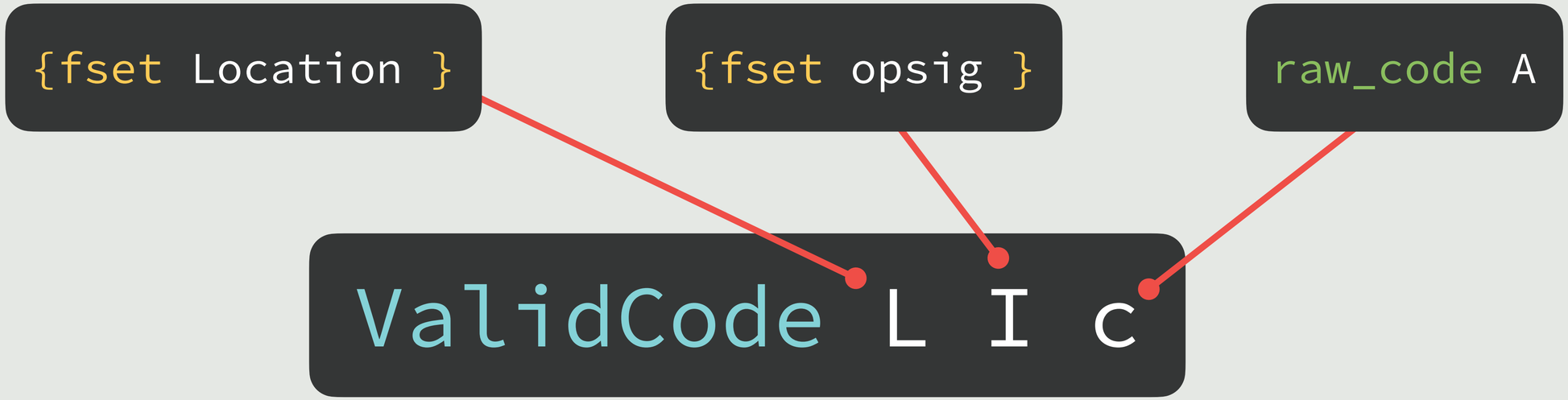
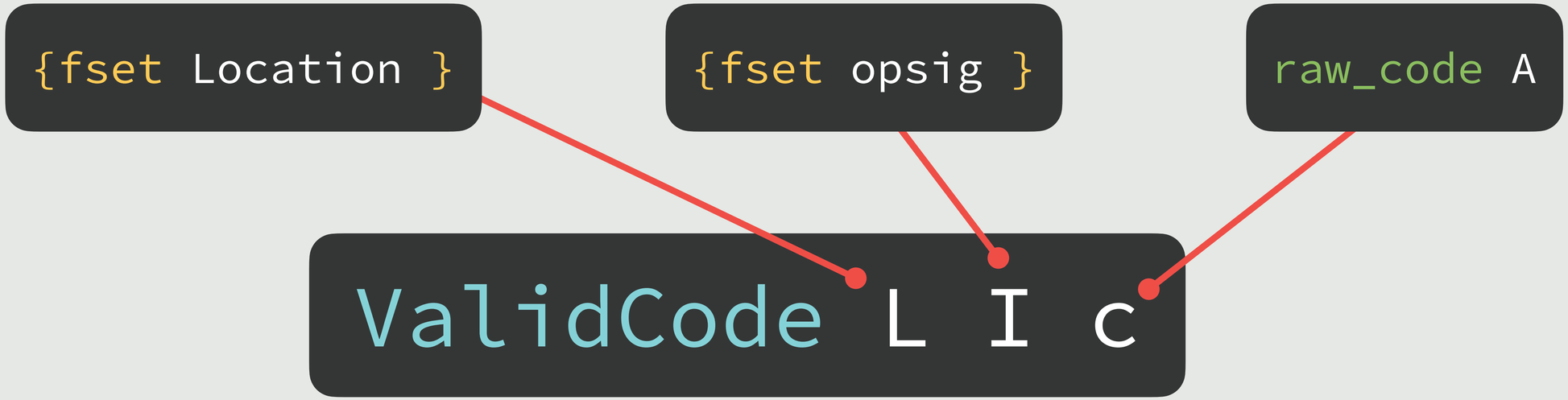



```
valid_ret : ∀ x, ValidCode L I (ret x)
```

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valid_ret : ∀ x, ValidCode L I (ret x)
```
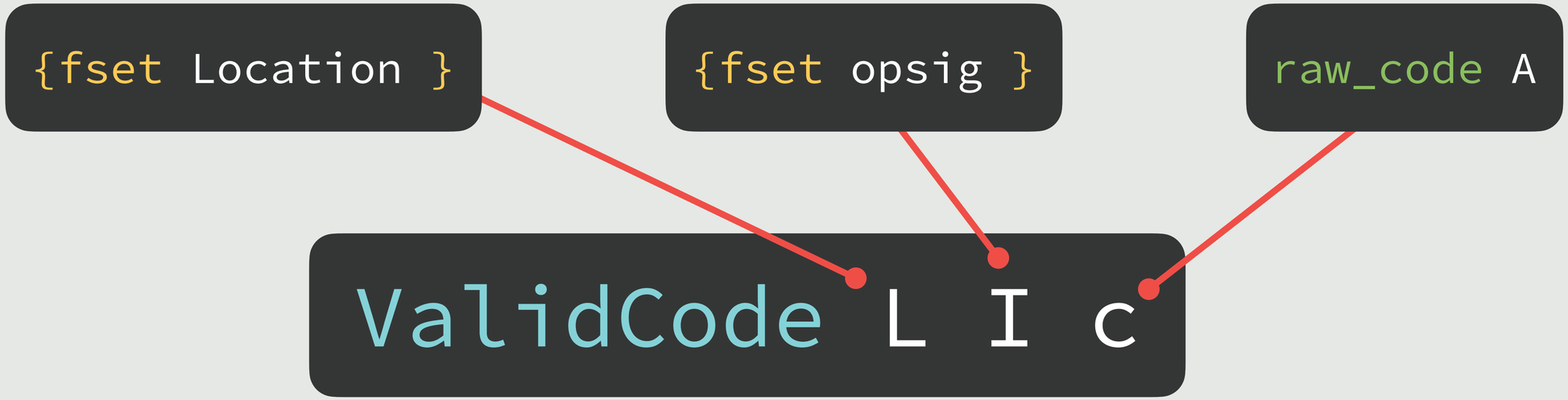

```
valid_getr :
  ∀ ℓ k,
    ℓ \in L →
    (∀ v, ValidCode L I (k v)) →
    ValidCode L I (x ← get ℓ ;; k x)
```

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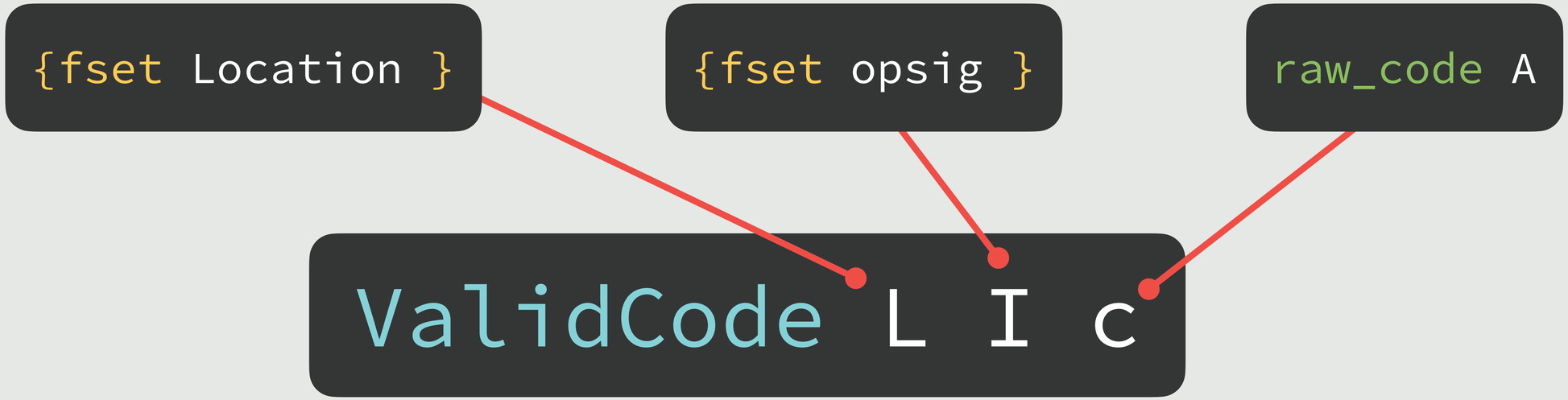


```
ValidCode L I (if b then c₀ else c₁)
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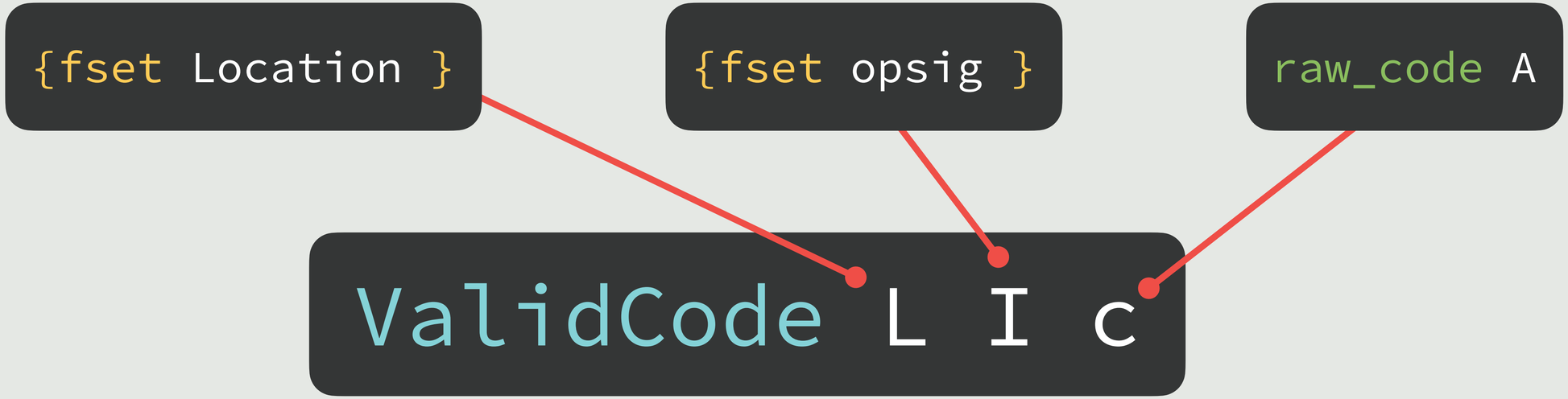


```
ValidCode L I (if b then c₀ else c₁)
```

```
#[export] Hint Extern 2 (ValidCode ?L ?I (match ?t with _ => _ end)) =>
  destruct t
  : typeclass_instances ssprove_valid_db.
```

```
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  Σ (S T : chUniverse), S → raw_code T.

Definition raw_package :=
  {fmap ident -> typed_raw_function }.
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Class ValidPackage L I E p :=
  ∀ o, o \in E →
    let '(id, (src, tgt)) := o in
    ∃ (f : src → raw_code tgt),
      p id = Some (src ; tgt ; f) ∧
      ∀ x, ValidCode L I (f x).
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```

```
Record package L I E := mkpackage {
  pack : raw_package ;
  pack_valid : ValidPackage L I E pack
}.
```

```
Definition IND_CPA⁰ :
  package [fset key_location ] [interface]
    [interface val #[ ENC ] : 'word → 'word × 'word ] :=
  [package
    def #[ ENC ] (m : 'word) : 'word × 'word
    {
      k ← get key_location ;;
      match k with
      | None =>
        k_val ← sample uniform i_key ;;
        put key_location := Some k_val ;;
        enc m k_val
      | Some k_val =>
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Package equivalence

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$\approx_{\varepsilon}$

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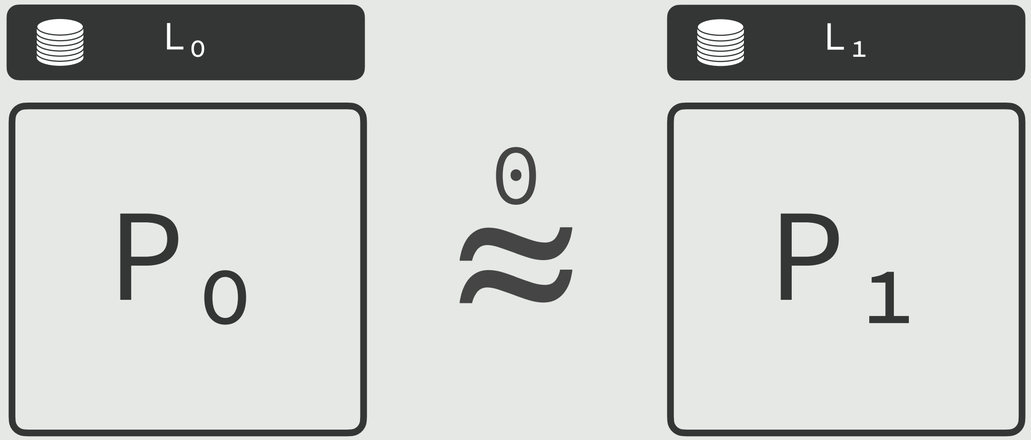
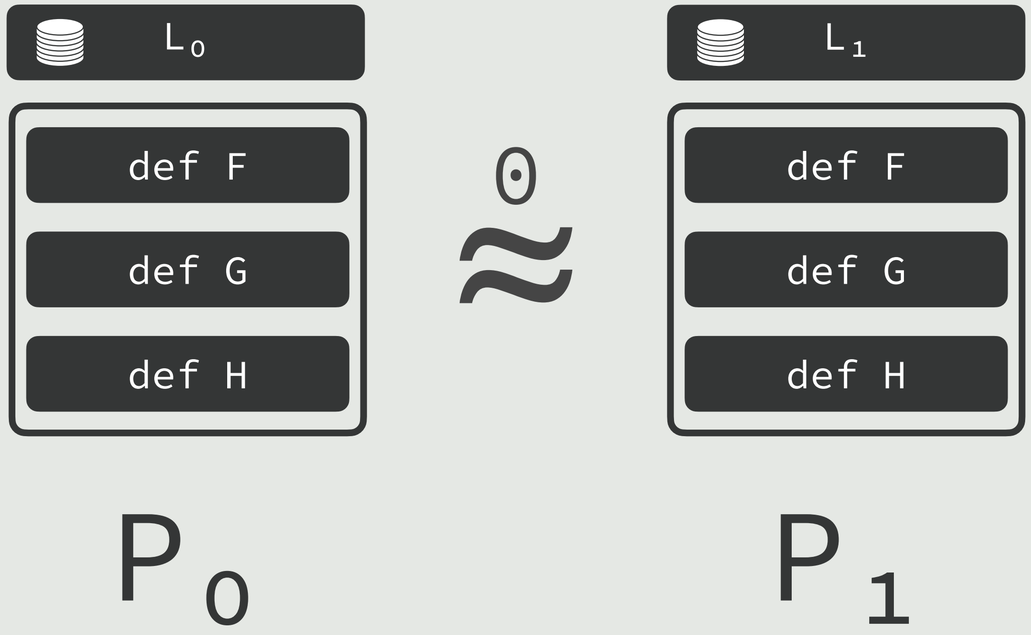


Package equivalence



If

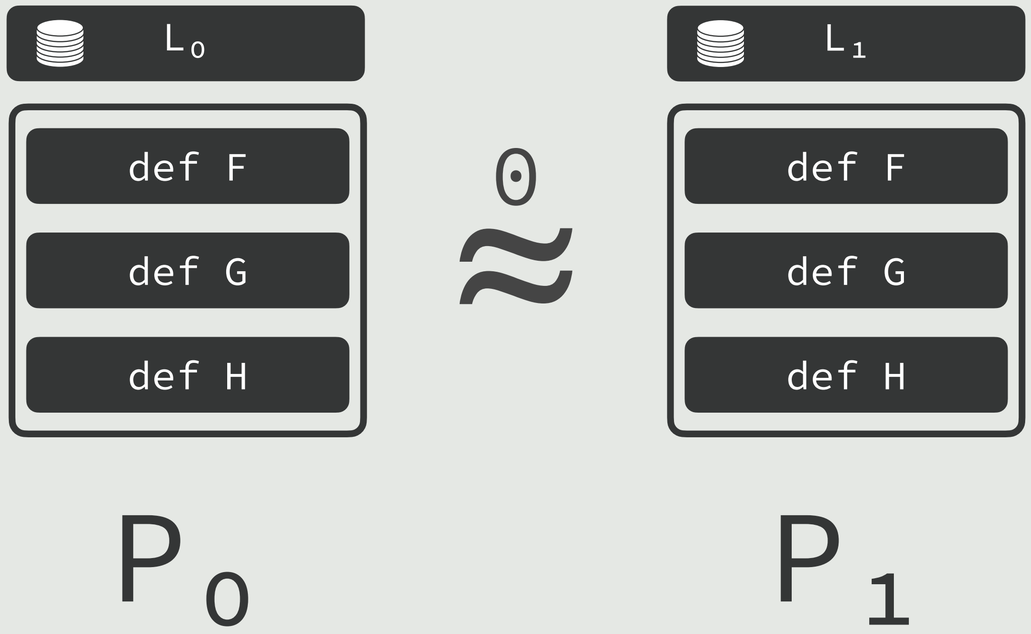
$$\forall\ x,\ \vdash P_0.F\ x \approx P_1.F\ x$$

$$\forall\ x,\ \vdash P_0.G\ x \approx P_1.G\ x$$

$$\forall\ x,\ \vdash P_0.H\ x \approx P_1.H\ x$$

modulo invariant ψ respecting/ignoring L_0, L_1

then



Probabilistic relational Hoare program logic

$$\forall\ x,\ \vdash\ P_0.F\ x \approx P_1.F\ x$$

modulo invariant ψ respecting/ignoring L_0, L_1

Probabilistic relational Hoare program logic

$$\forall\ x,\ \vdash\ \{\!|\ \mathtt{pre}\ |\!\}\ \mathtt{P}_0\mathtt{.F}\ x \approx \mathtt{P}_1\mathtt{.F}\ x\ \{\!|\ \mathtt{post}\ |\!\}$$

modulo invariant ψ respecting/ignoring $\mathtt{L}_0, \mathtt{L}_1$

Probabilistic relational Hoare program logic

$$\forall\ x,\ \vdash\ \{\!|\ \mathrm{pre}\ |\!\}\ P_0.F\ x \approx P_1.F\ x\ \{\!|\ \mathrm{post}\ |\!\}$$

$$\mathrm{pre} := \lambda\ '(s_0, s_1),\ \psi\ s_0\ s_1$$
$$\mathrm{post} := \lambda\ '(r_0, s_0)\ '(r_1, s_1),\ \psi\ s_0\ s_1 \wedge r_0 = r_1$$

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Probabilistic relational Hoare program logic

$$\forall\ x,\ \vdash\ \{\!|\ \mathit{pre}\ |\!\}\ P_0.F\ x \approx P_1.F\ x\ \{\!|\ \mathit{post}\ |\!\}$$

$$\begin{aligned}\mathit{pre}\ &:= \lambda\ '(s_0, s_1), \psi\ s_0\ s_1\\ \mathit{post}\ &:= \lambda\ '(r_0, s_0)\ '(r_1, s_1), \psi\ s_0\ s_1 \wedge r_0 = r_1\end{aligned}$$

ψ respecting/ignoring L_0, L_1

$$\psi\ \mathit{empty\_heap}\ \mathit{empty\_heap}$$

$$\begin{aligned}&\ell \setminus\!\mathrm{notin}\ L_0 \to \ell \setminus\!\mathrm{notin}\ L_1 \to \psi\ s_0\ s_1 \to\\ &\psi\ (\mathit{set\_heap}\ \ell\ v\ s_0)\ (\mathit{set\_heap}\ \ell\ v\ s_1)\end{aligned}$$

Probabilistic relational Hoare program logic

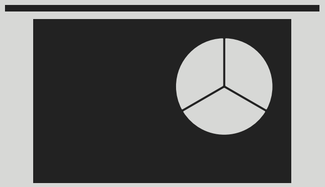
$$\vdash \{\!|\ \mathtt{pre}\ |\!\}\ \mathtt{c}_0 \approx \mathtt{c}_1\ \{\!|\ \mathtt{post}\ |\!\}$$

semantics with computation + specification monads

Probabilistic relational Hoare program logic

$$\vdash \{\!|\ \mathtt{pre}\ |\!\}\ \mathtt{c}_0 \approx \mathtt{c}_1\ \{\!|\ \mathtt{post}\ |\!\}$$

semantics with computation + specification monads



More details

Antoine’s talk at TYPES 2021

Examples

Synchronisation

Examples

Synchronisation

$$\vdash \{\!|\ \mathtt{pre}\ |\!\}\ x \leftarrow \mathtt{sample}\ D\ ;;\ k_0\ x \approx x \leftarrow \mathtt{sample}\ D\ ;;\ k_1\ x\ \{\!|\ \mathtt{post}\ |\!\}$$

Examples

Synchronisation

```
⊢ ⦃ pre ⦄ x ← sample D ;; k₀ x ≈ x ← sample D ;; k₁ x ⦃ post ⦄
```

```
ssprove_sync.
```

Examples

Synchronisation

```
⊢ ⦃ pre ⦄ x ← sample D ;; k₀ x ≈ x ← sample D ;; k₁ x ⦃ post ⦄
```

```
                                                    ssprove_sync.
```

```
∀ x, ⊢ ⦃ pre ⦄ k₀ x ≈ k₁ x ⦃ post ⦄
```

Examples

Synchronisation

Examples

Synchronisation

$$\vdash \{\!|\ \mathsf{pre}\ |\!\}\ x \leftarrow \mathsf{get}\ \ell\ ;;\ k_0\ x \approx x \leftarrow \mathsf{get}\ \ell\ ;;\ k_1\ x\ \{\!|\ \mathsf{post}\ |\!\}$$

Examples

Synchronisation

```
⊢ ⦃ pre ⦄ x ← get ℓ ;; k₀ x ≈ x ← get ℓ ;; k₁ x ⦃ post ⦄
```

```
ssprove_sync.
```

Examples

Synchronisation

```
⊢ ⦃ pre ⦄ x ← get ℓ ;; k₀ x ≈ x ← get ℓ ;; k₁ x ⦃ post ⦄
```

```
                                                    ssprove_sync.
```

```
∀ x, ⊢ ⦃ pre ⦄ k₀ x ≈ k₁ x ⦃ post ⦄
```

1.

```
get_pre_cond ℓ pre
```

2.

Examples

Synchronisation

```
⊢ ⦃ pre ⦄ x ← get ℓ ;; k₀ x ≈ x ← get ℓ ;; k₁ x ⦃ post ⦄
```

```
                                                    ssprove_sync.
```

```
∀ x, ⊢ ⦃ pre ⦄ k₀ x ≈ k₁ x ⦃ post ⦄
```

1.

```
get_pre_cond ℓ pre
```

2.

```
:= ∀ s₀ s₁, pre (s₀, s₁) → get_heap s₀ ℓ = get_heap s₁ ℓ
```

Examples

Synchronisation

```
⊢ ⦃ pre ⦄ x ← get ℓ ;; k₀ x ≈ x ← get ℓ ;; k₁ x ⦃ post ⦄
```

```
                                                    ssprove_sync.
```

```
∀ x, ⊢ ⦃ pre ⦄ k₀ x ≈ k₁ x ⦃ post ⦄
```

1.

```
get_pre_cond ℓ pre
```

2.

```
:= ∀ s₀ s₁, pre (s₀, s₁) → get_heap s₀ ℓ = get_heap s₁ ℓ
```

Examples

Swapping

```
⊢ ⦃ pre ⦄
  x ← get ℓ ;;
  y ← sample D ;;
  put ℓ := v ;;
  k x y
≈ c ⦃ post ⦄
```

Examples

Swapping

```
⊢ {| pre |}
0. x ← get ℓ ;;
1. y ← sample D ;;
2. put ℓ := v ;;
3. k x y
≈ c {| post |}
```

Examples

Swapping

```
⊢ ⦃ pre ⦄
0. x ← get ℓ ;;
1. y ← sample D ;;
2. put ℓ := v ;;
3. k x y
≈ c ⦃ post ⦄
```

```
ssprove_swap_lhs 1.
```

Examples

Swapping

```
⊢ ⦃ pre ⦄
0. x ← get ℓ ;;
2. put ℓ := v ;;
1. y ← sample D ;;
3. k x y
≈ c ⦃ post ⦄
```

```
ssprove_swap_lhs 1.
```

SSProve

SSP + $\vdash_{\approx}$ + 

Verified examples: PRF, ElGamal, KEM-DEM*

/SSProve/ssprove

links to videos in the README