

See the proof for Theorem `nat_to_binary_is_zero` in table 1

Table 1: Proof of Theorem `nat_to_binary_is_zero`

Next step in Coq	Proof situation
<i>Proof.</i>	$\vdash \text{forall } i : \text{nat}, \text{binary\_is\_zero } (\text{nat\_to\_binary } i) \rightarrow i = 0$
<i>intro i.</i>	$\boxed{i} : \text{nat} \vdash \text{binary\_is\_zero } (\text{nat\_to\_binary } i) \rightarrow i = 0$
<i>pattern i.</i>	$\vdash (\text{fun } n : \text{nat} \Rightarrow \text{binary\_is\_zero } (\text{nat\_to\_binary } n) \rightarrow n = 0) i$
<i>apply binary_induction; [tauto].</i>	$\vdash \text{forall } n : \text{nat}, n <> 0 \rightarrow (\text{binary\_is\_zero } (\text{nat\_to\_binary } n) \rightarrow n = 0) \rightarrow \text{binary\_is\_zero } (\text{nat\_to\_binary } (2 * n)) \rightarrow 2 * n = 0$ Hidden 1 goal(s)
<i>intros n H1 IH H2.</i>	$n : \text{nat} ; H1 : n <> 0 ; IH : \text{binary\_is\_zero } (\text{nat\_to\_binary } n) \rightarrow n = 0 ; H2 : \text{binary\_is\_zero } (\text{nat\_to\_binary } (2 * n)) \vdash 2 * n = 0$ Hidden 1 goal(s)
<i>rewrite IH; [tauto].</i>	$n : \text{nat} ; H1 : n <> 0 ; IH : \text{binary\_is\_zero } (\text{nat\_to\_binary } n) \rightarrow n = 0 ; H2 : \text{binary\_is\_zero } (\text{nat\_to\_binary } (2 * n)) \vdash \text{binary\_is\_zero } (\text{nat\_to\_binary } n)$ Hidden 1 goal(s)
<i>rewrite nat_to_binary_even in H2; [exact H1].</i>	$n : \text{nat} ; H1 : n <> 0 ; IH : \text{binary\_is\_zero } (\text{nat\_to\_binary } n) \rightarrow n = 0 ; H2 : \text{binary\_is\_zero } (\text{false} :: \text{nat\_to\_binary } n) \vdash \text{binary\_is\_zero } (\text{nat\_to\_binary } n)$ Hidden 1 goal(s)
<i>inversion H2.</i>	$n : \text{nat} ; H1 : n <> 0 ; IH : \text{binary\_is\_zero } (\text{nat\_to\_binary } n) \rightarrow n = 0 ; H2 : \text{binary\_is\_zero } (\text{false} :: \text{nat\_to\_binary } n) ; x : \text{bool} ; l : \text{list bool} ; H3 : \text{false} = \text{false} ; H4 : \text{Forall } (\text{eq false}) (\text{nat\_to\_binary } n) ; H : x = \text{false} ; H0 : l = \text{nat\_to\_binary } n \vdash \text{binary\_is\_zero } (\text{nat\_to\_binary } n)$ Hidden 1 goal(s)
<i>assumption.</i>	$\vdash \text{forall } n : \text{nat}, (\text{binary\_is\_zero } (\text{nat\_to\_binary } n) \rightarrow n = 0) \rightarrow \text{binary\_is\_zero } (\text{nat\_to\_binary } (2 * n + 1)) \rightarrow 2 * n + 1 = 0$
<i>intros n IH H1.</i>	$n : \text{nat} ; IH : \text{binary\_is\_zero } (\text{nat\_to\_binary } n) \rightarrow n = 0 ; H1 : \text{binary\_is\_zero } (\text{nat\_to\_binary } (2 * n + 1)) \vdash 2 * n + 1 = 0$
<i>rewrite nat_to_binary_odd in H1.</i>	$n : \text{nat} ; IH : \text{binary\_is\_zero } (\text{nat\_to\_binary } n) \rightarrow n = 0 ; \boxed{H1} : \text{binary\_is\_zero } (\text{true} :: \text{nat\_to\_binary } n) \vdash 2 * n + 1 = 0$
<i>inversion H1.</i>	$n : \text{nat} ; IH : \text{binary\_is\_zero } (\text{nat\_to\_binary } n) \rightarrow n = 0 ; x : \text{bool} ; l : \text{list bool} ; \boxed{H2} : \text{false} = \text{true} ; \boxed{H3} : \text{Forall } (\text{eq false}) (\text{nat\_to\_binary } n) ; \boxed{H} : x = \text{true} ; H0 : l = \text{nat\_to\_binary } n \vdash 2 * n + 1 = 0$
<i>discriminate.</i>	Proof finished by Qed

End of proof of Theorem `nat_to_binary_is_zero`